From fluctuating gravitons to Lorentzian quantum gravity

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Path integral for metric quantum gravity

- Assumptions
  - Metric carries fundamental degrees of freedom
  - Diffeomorphism invariance

- Path integral

\[
\int \mathcal{D}\hat{g}_{\mu\nu} e^{-S[\hat{g}]} \quad \text{or} \quad \int \mathcal{D}\hat{g}_{\mu\nu} e^{iS[\hat{g}]}
\]
Path integral for metric quantum gravity

- Assumptions
  - Metric carries fundamental degrees of freedom
  - Diffeomorphism invariance

- Path integral with gauge fixing, sources

\[ Z[\bar{g}, J] \sim \int \mathcal{D}[\hat{h}_{\mu\nu}] e^{-S[\bar{g} + \hat{h}]} - S_{gf}[\bar{g}, \hat{h}] - S_{gh}[\bar{g}, \hat{h}, \hat{c}, \hat{\bar{c}}] + \int_x \sqrt{\bar{g}} J^{\mu\nu}(x) \hat{h}_{\mu\nu}(x) \]

- Metric split \( g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \) required by gauge fixing and regulator

- Methods: Perturbation theory, lattice, functional methods, ...
Einstein-Hilbert action

\[ S_{EH} = \frac{1}{16\pi G_N} \int_x \sqrt{\det g_{\mu\nu}} (2\Lambda - R(g_{\mu\nu})) \]

- Perturbatively non-renormalisable: \([G_N] = -2\)
- Need infinitely many counter terms: No predictivity

\[ [\text{'t Hooft, Veltmann '74; Goroff, Sagnotti '85}] \]
Perturbative quantum gravity

Einstein-Hilbert action

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- Perturbatively non-renormalisable: \([G_N] = -2\)
- Need infinitely many counter terms: No predictivity \([\text{\textquoteleft t Hooft, Veltmann '74; Goroff, Sagnotti '85}]\)

Higher-derivative action

\[ S_{HD} = S_{EH} + \int \sqrt{\det g_{\mu\nu}} \left( \frac{1}{2\lambda} C_{\mu\nu\rho\sigma}^2 - \frac{\omega}{3\lambda} R^2 \right) \]

- Perturbatively renormalisable: \([\omega] = [\lambda] = 0\)
- Non-unitary propagator \([\text{Stelle '74}]\)

Graviton propagator

\[ G_{\text{graviton}} \sim \frac{1}{p^2 + p^4/M_{\text{Pl}}^2} = \frac{1}{p^2} - \frac{1}{M_{\text{Pl}}^2 + p^2} \]
Non-perturbative tool: The Functional Renormalisation Group

**Non-perturbative renormalisation group equation** [Wetterich '93]

\[ k \partial_k \Gamma_k = \frac{1}{2} \text{Tr} \left[ \frac{1}{\Gamma_k^{(2)} + R_k} k \partial_k R_k \right] \]

\( R_k \) = regulator function

\( \Gamma_k \) = scale-dependent effective action

Interpolation between

- bare action / UV FP
- quantum effective action \( \Gamma \)
- Wilsonian integrating out of momentum modes

[Image: Gies '06]
The Quantum Effective Action

- Legendre transform of the logarithm of the generating functional

\[ \Gamma[\phi] = \sup_J \left\{ \int_x J(x) \phi(x) - \ln Z[J] \right\} \]

- Generates 1PI correlation functions and full quantum physics at tree level

Vertices from classical action $S$
No quantum effects

Vertices from quantum effective action $\Gamma$
Includes all quantum effects
Asymptotically safe quantum gravity

QG could be non-perturbatively renormalisable via an interacting UV FP

\[ S_{EH} = \frac{1}{16\pi G_N} \int x \sqrt{g} (2\Lambda - R) \]

[Weinberg '76]

\[ S_{EH} = \frac{1}{16\pi G_N} \int x \sqrt{g} (2\Lambda - R) \]

[Reuter '96; Reuter, Saueressig '01]

[MR '20]

Predictivity: number of free parameters = dimension of UV critical hypersurface

Unitarity: Positivity and finiteness of spectral functions and scattering amplitudes

[Bonanno, Denz, Pawlowski, MR '21; Fehre, Litim, Pawlowski, MR '21; ...]
Asymptotically safe quantum gravity

QG could be non-perturbatively renormalisable via an interacting UV FP

\[ S_{\text{EH}} = \frac{1}{16\pi G_N} \int_x \sqrt{g} \ (2\Lambda - R) \]

[Weinberg '76]

\[ \sim k^2 \]

\[ \sim k^{-2} \]

[Reuter '96; Reuter, Saueressig '01]

Predictivity: number of free parameters = dimension of UV critical hypersurface

[Denz, Pawlowski, MR '16; Falls, Ohta, Percacci '20; Kluth, Litim '20; Knorr '21; ...]

Unitarity: Positivity and finiteness of spectral functions and scattering amplitudes

[Bonanno, Denz, Pawlowski, MR '21; Fehre, Litim, Pawlowski, MR '21; ...]
Why it could work

- Enhanced symmetry at high energies: quantum scale invariance
- Perturbative UV fixed point in $2 + \varepsilon$ dimensions

$$\beta_g = \varepsilon g - bg^2$$

with

$$b = \frac{2}{3} \left( 19 + 9N_v - \frac{1}{2}N_f - N_s \right)$$

[Capper, Duff '74; ...]
[Review: Martini, Vacca, Zanusso '22]
Expansion in curvature invariants

\[ \Lambda \]

\[ R \]

\[ R^2 \quad R_{\mu \nu} R^{\mu \nu} \quad C_{\mu \nu \rho \sigma} C_{\mu \nu \rho \sigma} \]

\[ R^3 \quad R \nabla^2 R \quad C_{\mu \nu \rho \sigma} \nabla^2 C_{\mu \nu \rho \sigma} \quad C_{\mu \nu} C_{\kappa \lambda} C_{\kappa \lambda} C_{\rho \sigma} \]

\[ R^4 \quad R \nabla^4 R \quad C_{\mu \nu \rho \sigma} \nabla^4 C_{\mu \nu \rho \sigma} \]

\[ \vdots \quad \vdots \]

- Need to include all operators compatible with symmetry
- Only \( \Lambda \), \( R \), and \( R^2 \) are relevant

[Gies, Knorr, Lippoldt, Saueressig '16, Falls, Litim, Schröder '18; Knorr, Ripken, Saueressig '19; Kluth, Litim '20; Falls, Ohta, Percacci '20; Knorr '21; Baldazzi, Falls, Kluth, Knorr '23; …]
Fluctuation approach – expansion in momentum dependent graviton correlations

- Treat $\bar{g}$ and $h$ independently, resolve fluctuation correlation functions
  \[
  \Gamma_k[\bar{g}, h] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \Gamma_k^{(0, h_{a_1} ... h_{a_n})}[\bar{g}, 0] \cdot h_{a_1} \ldots h_{a_n}
  \]

- Flat background $\bar{g} = \delta$ allows for momentum-space techniques

\[
\begin{align*}
\partial_t \Gamma_k &= \frac{1}{2} \quad - \\
\partial_t \Gamma_k^{(2h)} &= -\frac{1}{2} + -2 \\
\partial_t \Gamma_k^{(3h)} &= -\frac{1}{2} + 3 - 3 + 6 \\
\partial_t \Gamma_k^{(4h)} &= -\frac{1}{2} + 3 + 4 - 6 - 12 + 12 - 24
\end{align*}
\]

[Christiansen, Knorr, Meibohm, Pawlowski, MR ’15; Denz, Pawlowski, MR ’16; Pawlowski, MR ’20; ’23; ...]
Fluctuation approach – expansion in momentum dependent graviton correlations

- Treat $\bar{g}$ and $h$ independently, resolve fluctuation correlation functions

$$\Gamma_k[\bar{g}, h] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \Gamma_k^{(0, h_{a_1} \ldots h_{a_n})}[\bar{g}, 0] \cdot h_{a_1} \ldots h_{a_n}$$

- Flat background $\bar{g} = \delta$ allows for momentum-space techniques

- Extremely large tensor basis

<table>
<thead>
<tr>
<th>Basis elements</th>
<th>TT-modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma^{(2h)}$</td>
<td>5</td>
</tr>
<tr>
<td>$\Gamma^{(3h)}$</td>
<td>33</td>
</tr>
<tr>
<td>$\Gamma^{(4h)}$</td>
<td>334</td>
</tr>
</tbody>
</table>

- Need careful projection on elements for universal one-loop beta functions of Stelle gravity

[MR (in prep)]
- Includes the running up to the transverse-traceless four-graviton vertex

- Three relevant directions: $\Lambda$, $R$, and $R^2$; $C_{\mu\nu\rho\sigma}^2$ irrelevant
Momentum dependent correlation functions

- Momentum dependent correlation functions integrated to \( k = 0 \)
- RG scale and momentum dependence agree qualitatively
Gravity-matter systems
Question 1: For which matter content does the UV fixed point exist?

Scalars

Fermions

Yang-Mills

Fixed point exists for (at least) SM matter

[Eichhorn, Labus, Pawlowski, MR '18 ]

[Eichhorn, Lippoldt, Schiffer '18]

[Christiansen, Litim, Pawlowski, MR '17]
Question 2: Does gravity help with Landau poles in the matter sector?
Avoiding Landau poles

Simple parameterisation of $U(1)$ beta function

$$\beta_{g_1} = \beta_{g_1, \text{matter}} - f_g g_1$$

[Eichhorn, Versteegen '17]

![Graph showing $g$ vs. TeV and $M_{Pl}$](image)
Avoiding Landau poles

Simple parameterisation of $U(1)$ beta function

$$\beta_{g_1} = \beta_{g_1, \text{matter}} - f_g g_1$$

[Eichhorn, Versteegen '17]

The Gaussian fixed point is

- relevant for gauge couplings
  - [Christiansen, Litim, Pawlowski, MR '17, Pastor-Gutiérrez, Pawlowski, MR '22, ...]
- irrelevant for quartic scalar couplings
  - [Eichhorn, Hamada, Lumma, Yamada '17; Pawlowski, MR, Wetterich, Yamada '18, ...]
- unclear for Yukawa couplings
  - [Eichhorn, Held '17; Pastor-Gutiérrez, Pawlowski, MR '22, ...]
• Gravity makes matter couplings asymptotically free
• Assumption: Yukawa coupling relevant at Gaussian fixed point

[Pastor-Gutiérrez, Pawlowski, MR '22]
Higgs vs top mass in the ASSM

- Predicted Higgs mass of 125 GeV before measurement
  [Shaposhnikov, Wetterich '12]

- Small mismatch between predicted and measured Higgs-top mass ratio in pure SM

- Can be fixed with BSM physics, e.g., dark matter
  [Reichert, Smirnov '19]

[10x237]Higgs vs top mass in the ASSM

[162x162]1% 2% 3% 4% 5%

[225x173]•

[235x173]Predicted Higgs mass of 125 GeV before measurement

[225x128]•

[235x128]Small mismatch between predicted and measured Higgs-top mass ratio in pure SM

[332x159][Shaposhnikov, Wetterich '12]

[354x70][Reichert, Smirnov '19]

[46x80][Pastor-Gutiérrez, Pawlowski, MR '22]

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Lorentzian computations
Towards testing Unitarity

A unitary theory requires

- Well-behaved propagators without ghost or tachyonic instabilities
- Bounds on scattering amplitudes, e.g., violated by GR

\[ h_{\text{GR}} \propto (p_1 + p_2)^2 \]

Need access to correlation functions on Lorentzian signature at time-like momenta
Källén-Lehmann spectral representation

\[ G(q^2) = \int_0^\infty \frac{d\lambda^2}{\pi} \frac{\rho(\lambda^2)}{q^2 - \lambda^2} \]

with

\[ \rho(\omega^2) = -\lim_{\varepsilon \to 0} \text{Im} \ G(\omega^2 + i\varepsilon) \]
Classical graviton spectral function

Einstein-Hilbert action: \( S_{EH} = \frac{1}{16\pi G_N} \int x \sqrt{g} (2\Lambda - R) \)

Flat Minkowski background: \( g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \)

\[ G_{hh}(p^2) \sim \frac{1}{p^2} \]

\[ \rho_h(\omega^2) \sim \delta(\omega^2) \]
Classical graviton spectral function

Higher-derivative action: \( S_{HD} = S_{EH} + \int x \sqrt{g} \left( aR^2 + bC^2_{\mu\nu\rho\sigma} \right) \)

\[
G_{hh}(p^2) \sim \frac{1}{p^2} - \frac{1}{M_{Pl}^2 + p^2}
\]

\[
\rho_h(\omega^2) \sim \delta(\omega^2) - \delta(\omega^2 - M_{Pl}^2)
\]
One-loop effective action: \( \Gamma_{\text{1-loop}} = S_{\text{EH}} + \int_x \sqrt{g} \left( c_1 R \ln(\Box) R + c_2 C_{\mu\nu\rho\sigma} \ln(\Box) C^{\mu\nu\rho\sigma} \right) + \ldots \)

Gauge-fixing \( S_{\text{gf}} = \frac{1}{\alpha} \int F_\mu^2 \) with \( F_\mu = \nabla^\nu h_{\mu\nu} - \frac{1+\beta}{4} \nabla_\mu h^{\nu\nu} \)

\[
G_{hh}(p^2) \sim \frac{1}{p^2 + \ln(p^2)\rho^4}
\]

\[
\rho_h(\omega^2) \sim \delta(\omega^2) + \text{const.} + \ldots
\]

[Pawlowski, MR ’23]
Regulator $R_k = k^2 r_k(x)$

- $r_k = r_k(p^2/k^2)$ breaks causality
- $r_k = r_k(\bar{p}^2/k^2)$ breaks Lorentz invariance
- $r_k = 1$ provides no UV regularisation
• Callan-Symanzik cutoff uniquely preserves causality and Lorentz invariance

\[ R_k = Z_\phi k^2 \]

• Finite flow equation with counterterms

\[ \partial_t \Gamma_k = \frac{1}{2} \text{Tr} G_k \partial_t R_k - \partial_t S_{ct,k} \]

• Dimensional regularisation of UV divergences in \( d = 4 - \varepsilon \) possible

• Finitely many counter terms if gravity is asymptotically safe
Lorentzian setup

- Expansion about flat Minkowski background

- Direct flow of \( \rho_h \) with \( m_h^2 = k^2(1 + \mu) \) and \( Z_h = Z_h(p^2 = -m_h^2) \)

\[
\rho_h = \frac{1}{Z_h} \left[ 2\pi \delta(\lambda^2 - m_h^2) + \theta(\lambda^2 - 4m_h^2) f_h(\lambda) \right]
\]

- Use \( \rho_h \) in flow diagrams

\[
\partial_t \rho_h \propto \begin{array}{c}
\text{Diagram}
\end{array} + \ldots \quad \text{with} \quad G_h(q^2) = \int_0^\infty \frac{d\lambda^2}{\pi} \frac{\rho_h(\lambda^2)}{q^2 - \lambda^2}
\]
Lorentzian UV-IR trajectories

\[(g, \eta_h, \mu)|_* = (1.06, 0.96, -0.34)\]
\[\theta = 2.49 \pm 3.17i\]

\[G_N(k) = g(k)/k^2 \xrightarrow{k \to 0} G_N\]
\[-2\Lambda(k) = k^2 \mu(k) \xrightarrow{k \to 0} -2\Lambda = 0\]
• Massless graviton delta-peak with multi-graviton continuum
• No ghosts and no tachyons → no indications for unitarity violation
• Good agreement with reconstruction results and EFT

[Fehre, Litim, Pawlowski, MR '21]

[Bonanno, Denz, Pawlowski, MR '21]
Towards graviton-mediated scattering cross-sections

\[ e^- e^+ + \mu^- \mu^+ = \phi_{\text{matter}} + h + e^- e^+ + \mu^- \mu^+ \]
Towards graviton-mediated scattering cross-sections

\[ e^- + \mu^- + = e^- + \mu^- + \]

\[ 10^{-45} \quad 10^{-40} \quad 10^{-35} \quad 10^{-30} \]

\[ 10^{16} \quad 10^{17} \quad 10^{18} \quad 10^{19} \quad 10^{20} \quad 10^{21} \]

[Pastor-Gutiérrez, Pawlowski, MR, Ruisi (in prep)]
Summary

- Asymptotically safety is a strong contender for the fundamental theory of quantum gravity
- Predictive theory that can encapsulate the Standard Model
- Direct Lorentzian computation of graviton spectral function with spectral fRG
- Well-behaved spectral function without ghost or tachyonic instabilities
- Key step towards scattering processes and unitarity
Asymptotically safety is a strong contender for the fundamental theory of quantum gravity

Predictive theory that can encapsulate the Standard Model

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Thank you for your attention!