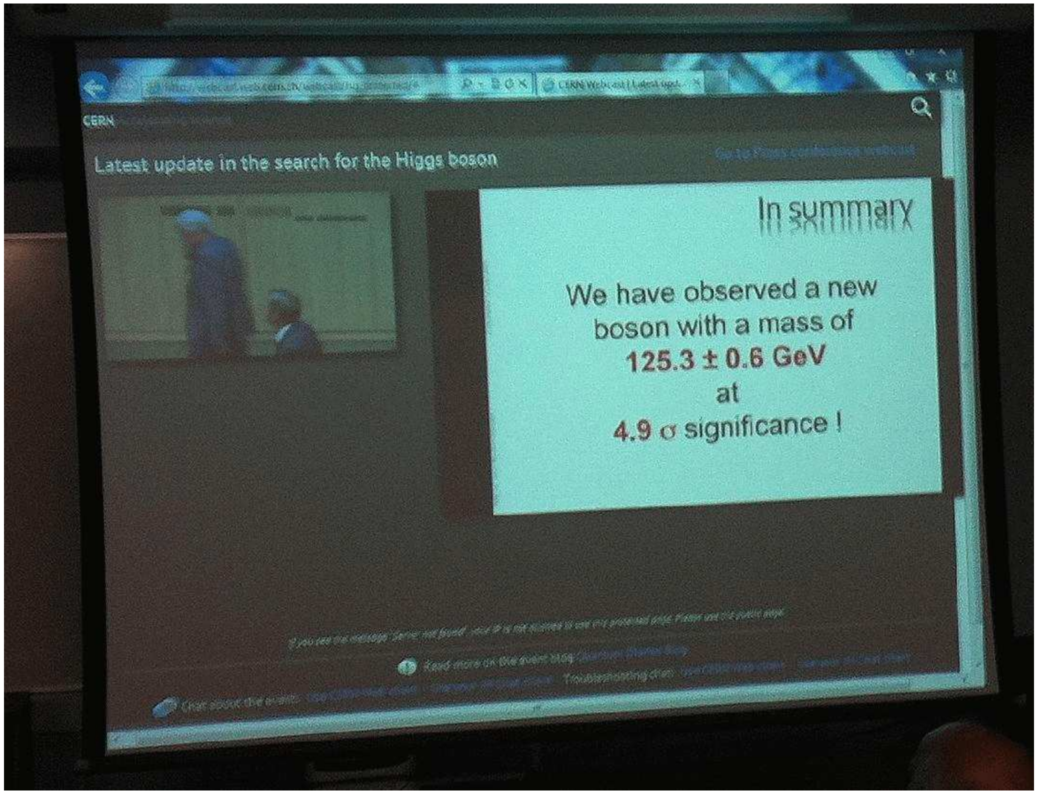
String Phenomenology: past, present and future

Progress Report: The early years 1989–2013

With: Dimitri Nanopoulos, Kajia Yuan, Edi Halyo, Ben Grinstein, Keith Dienes, Claudio Coriano, Sangyeon Chang, Marco Matone, Jogesh Pati, John Ellis, Gerald Cleaver, Per Berglund, Zongan Qiu, Maxim Pospelov, Richard Garavuso, Jose Isidro, Alessandro Cafarella, Sander Nooij, John Rizos, Costas Kounnas, Cristina Timirgaziu, Elisa Manno, Marco Guzzi, Ben Assel, Kyriakos Christodoulides, Mirian Tsulaia, Thomas Mohaupt, Radu Tatar, William Walters, Viraf Mehta, Hassan Sonmez, Laura Bernard, Ivan Glasser, ... +

String Phenomenology Group Meeting, Liverpool, 5 November 2013

... Before



4 July 2012

After ...

DATA $\longrightarrow$ STANDARD MODEL

$SU(3) \times SU(2) \times U(1)_Y \longrightarrow SU(5) \longrightarrow SO(10)$

$$\left[\begin{pmatrix} \nu \\ e \end{pmatrix} + D^c_L \right] + \left[U^c_L + \begin{pmatrix} u \\ d \end{pmatrix} + E^c_L \right] + N^c_L$$

$$\bar{5} \quad + \quad 10 \quad 1 \qquad \overline{} \quad 16$$

STANDARD MODEL -> UNIFICATION

ADDITIONAL EVIDENCE:

Logarithmic running, proton longevity, neutrino masses

PRIMARY GUIDES:

3 generations

SO(10) embedding

Realistic free fermionic models

'Phenomenology of the Standard Model and string unification'

- Top quark mass $\sim$ 175–180GeV — PLB 274 (1992) 47
- Generation mass hierarchy — NPB 407 (1993) 57
- CKM mixing — NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism — PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification — NPB 457 (1995) 409 (with Dienes)
- Proton stability — NPB 428 (1994) 111
- Squark degeneracy — NPB 526 (1998) 21 (with Pati)
- Minimal Superstring Standard Model — PLB 455 (1999) 135 (with Cleaver & Nanopoulos)
- Moduli fixing — NPB 728 (2005) 83
- Exophobia — PLB 683 (2010) 306 (with Assel, Christodoulides, Kounnas & Rizos)

Other approaches

Geometrical

Greene, Kirklin, Miron, Ross (1987)
Donagi, Ovrut, Pantev, Waldram (1999)
Blumenhagen, Moster, Reinbacher, Weigand (2006)
Heckman, Vafa (2008)
....

Orbifolds

Ibanez, Nilles, Quevedo (1987)
Bailin, Love, Thomas (1987)
Kobayashi, Raby, Zhang (2004)
Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter (2007)
Blaszczyk, Groot–Nibbelink, Ruehle, Trapletti, Vaudrevange (2010)
....

Other CFTs

Gepner (1987)
Schellekens, Yankielowicz (1989)
Gato–Rivera, Schellekens (2009)
....

Orientifolds

Cvetic, Shiu, Uranga (2001)
Ibanez, Marchesano, Rabadan (2001)
Kiristis, Schellekens, Tsulaia (2008)
....

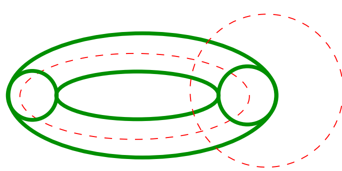
Free Fermionic Construction

Left-Movers: $\psi^{\mu}_{1,2}, \quad \chi_i, \quad y_i, \; \omega_i \quad (i = 1, \cdots, 6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \;,\; \bar{\omega}_i & i = 1, \cdots, 6 \\ \bar{\eta}_i & i = 1, 2, 3 \\ \bar{\psi}_{1,\cdots,5} & \\ \bar{\phi}_{1,\cdots,8} & \end{cases}$$

$$V \longrightarrow V$$



$$f \longrightarrow -e^{i\pi\alpha(f)} f$$

$$Z = \sum_{\substack{all\ spin \\ structures}} c\binom{\vec{\alpha}}{\vec{\beta}} \, Z\binom{\vec{\alpha}}{\vec{\beta}}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

The NAHE set:

$$1 \; = \; \{\psi^\mu, \; \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S \; = \; \{\psi^\mu, \chi^{1,\dots,6}\}, \qquad N = 4 \text{ Vacua}$$

$$b_1 \; = \; \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N = 4 \rightarrow N = 2$$

$$b_2 \; = \; \{\chi^{12}, \chi^{56}, y^{12}, \omega^{56} | \bar{y}^{12}, \bar{\omega}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \rightarrow N = 1$$

$$b_3 \; = \; \{\chi^{12}, \chi^{34}, \omega^{12}, \omega^{34} | \bar{\omega}^{12}, \bar{\omega}^{34}, \bar{\eta}^3, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \rightarrow N = 1$$

$Z_2 \times Z_2$ orbifold compactification

$\Longrightarrow$ Gauge group $SO(10) \times SO(6)^{1,2,3} \times E_8$

beyond the NAHE set Add $\{\alpha, \beta, \gamma\}$

number of generations is reduced to three

$$SO(10) \quad \longrightarrow \quad SU(3) \times SU(2) \times U(1)_{T_{3_R}} \times U(1)_{B-L}$$

$$U(1)_Y \; = \; \frac{1}{2}(B-L) \; + \; T_{3_R} \; \in \; SO(10) \; !$$

$$SO(6)^{1,2,3} \quad \longrightarrow \quad U(1)^{1,2,3} \; \times \; U(1)^{1,2,3}$$

STRING DERIVED STANDARD-LIKE MODEL (PLB278)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,...,6}$	$\bar{y}^{3,...,6}$	$y^{1,2},\omega^{5,6}$	$\bar{y}^{1,2},\bar{\omega}^{5,6}$	$\omega^{1,...,4}$	$\bar{\omega}^{1,...,4}$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
1	1	1	1	1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0	0	0	0,...,0
b_1	1	1	0	0	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1	0	0	0,...,0
b_2	1	0	1	0	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0	1,...,1	0	1	0	0,...,0
b_3	1	0	0	1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1	1,...,1	0	0	1	0,...,0

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0	0	0	0	1 1
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0	0	0	0	1 1
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1

Asymmetric $BC \Rightarrow$ all untwisted moduli are projected out!

all $y_i \omega_i \bar{y}_i \bar{\omega}_i$ are disallowed

can be translated to asymmetric bosonic identifications

$$X_L + X_R \rightarrow X_L - X_R$$

moduli fixed at enhanced symmetry point

Calculation of Mass Terms

nonvanishing correlators

$$\langle V_1^f V_2^f V_3^b \cdot \cdots V_N^b \rangle$$

gauge & string invariant

"anomalous" $U(1)_A$

$$\text{Tr}Q_A \neq 0 \Rightarrow \quad D_A = 0 = A + \sum Q_k^A |\langle \phi_k \rangle|^2$$

$$D_j \ = 0 = \qquad \sum Q_k^j |\langle \phi_k \rangle|^2$$

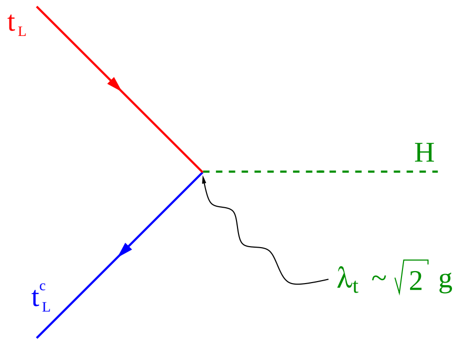
$$\langle W \rangle = \langle \frac{\partial W_N}{\partial \eta_i} \rangle = 0 \quad N = 3 \cdots \cdot$$

Supersymmetric vacuum $\langle F \rangle = \langle D \rangle = 0.$

nonrenormalizable terms $\longrightarrow$ effective renormalizable operators

$$V_1^f V_2^f V_3^b \cdot \cdots V_N^b \ \rightarrow \ V_1^f V_2^f V_3^b \frac{\langle V_4^b \cdots V_N^b \rangle}{M^{N-3}}$$

Top Quark Mass Prediction



Only $\lambda_t = \langle Q t^c_L H \rangle = \sqrt{2} g$ at $N = 3$

mass of lighter quarks and leptons $\rightarrow$ nonrenormalizable terms

$$\longrightarrow \qquad \lambda_b \;=\; \lambda_\tau \;=\; 0.35 g^3 \;\sim\; \frac{1}{8}\lambda_t$$

Evolve λ_t , λ_b to low energies

$$m_t \;=\; \lambda_t v_1 \;=\; \lambda_t \frac{v_0}{\sqrt{2}} \sin\beta \qquad\qquad m_b \;=\; \lambda_b v_2 \;=\; \lambda_b \frac{v_0}{\sqrt{2}} \cos\beta$$

where $v_0 = \dfrac{2 m_W}{g_2(M_Z)} = 246 \mathrm{GeV}$ and $v_1^2 + v_2^2 = \dfrac{v_0^2}{2}$

$$m_t \;=\; \lambda_t(m_t)\, \frac{v_0}{\sqrt{2}}\, \frac{\tan\beta}{(1 + \tan^2\beta)^{\frac{1}{2}}} \Longrightarrow$$

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PHYSICS LETTERS B

Hierarchical top–bottom mass relation in a superstring derived standard-like model

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I propose a mechanism in a class of superstring standard-like models which explains the mass hierarchy between the top and bottom quarks. At the trilinear level of the superpotential only the top quark gets a nonvanishing mass term while the bottom quarks and tau lepton mass terms are obtained from nonrenormalizable terms. I construct a model which realized this mechanism. In this model the bottom quark and tau lepton Yukawa couplings are obtained from quartic order terms. I show that $\lambda_b = \lambda_\tau \sim \frac{1}{2}\lambda_t$ at the unification scale. A naive estimate yields $m_t \sim 175$–180 GeV.

One of the unresolved puzzles of the standard model is the mass splitting between the top quark and the lighter ‌juarks and leptons. Especially difficult to understand within the context of the standard model is the big split-ing in the heaviest generation. Experimental limits [1] indicate the top mass to be above 80 GeV, while the ottom and tau lepton masses are found at 5 GeV and 1.78 GeV respectively. Possible extensions to the standard nodel are grand unified theories. Although the main prediction of GUTs, proton decay, has not yet been ob-erved, calculations of $\sin^2\theta_W$ and of the mass ratio m_b/m_τ support their validity. Recent calculations seem to upport supersymmetric GUTs versus nonsupersymmetric ones [2]. In spite of the success of SUSY GUTs in onfronting LEP data [2], an understanding of the mass splitting between the top quark and the lighter quarks nd leptons is still lacking. The next level in which such an understanding may be developed is in the context of uperstring theory [3].

Cabibbo mixing PLB 307 (1993) 305

Find anomaly free solution

$$M_d \sim \begin{pmatrix} \epsilon & \frac{V_2 \bar{V}_3 \Phi_{\alpha\beta}}{M^3} & 0 \\ \frac{V_2 \bar{V}_3 \Phi_{\alpha\beta} \xi_1}{M^4} & \frac{\bar{\Phi}_2^- \xi_1}{M^2} & 0 \\ 0 & 0 & \frac{\Phi_1^+ \xi_2}{M^2} \end{pmatrix} v_2,$$

$$\epsilon < 10^{-8} \qquad \frac{V_2 \bar{V}_3 \Phi_{\alpha\beta}}{M^3} = \frac{\sqrt{5}\, g^6}{64\, \pi^3} \approx 2 - 3 \times 10^{-4}.$$

$$\Rightarrow \quad |V| \sim \begin{pmatrix} 0.98 & 0.2 & 0 \\ 0.2 & 0.98 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Three generation mixing $\longrightarrow$ NPB 416 (1994) 63 $\quad |J| \sim 10^{-6}$

Cleaver, Faraggi, Manno, Timirgaziu, PRD 78 (2008) 046009

Classification of F and D flat directions in FMT reduced Higgs model

No D flat direction which is F-flat up to order eight in the superpotential

no stringent flat directions to all orders

Suggesting no supersymmetric flat directions in this model (class of models)

implying no supersymmetric moduli

only remaining perturbative moduli is the dilaton

quasi–realistic model: SLM; 3 gen; $SO(10)$ embed; Higgs & $\lambda_t \sim 1$; ...

vanishing one–loop partition function, perturbatively broken SUSY

Fixed geometrical, twisted and SUSY moduli

Cleaver $etal$, $SO(10)$ and FSU5 analysis $->$ stringent flat directions

Classification of fermionic $Z_2 \times Z_2$ orbifolds

Basis vectors:

$$1 = \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1, \dots, 6,$$

$N = 4$ Vacua

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\},$$

$N = 4 \to N = 2$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\},$$

$N = 2 \to N = 1$

$$\alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}$$

& Gauge group $SO(6) \times SO(4) \times U(1)^3 \times \text{hidden}$

Independent phases $c\binom{v_i}{v_j} = \exp[i\pi(v_i|v_j)]$: upper block

$$
\begin{array}{c}
 \\ 1 \\ S \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ z_1 \\ z_2 \\ b_1 \\ b_2 \\ \alpha
\end{array}
\begin{array}{ccccccccccccc}
1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2 & \alpha
\end{array}
$$

$$
\begin{array}{c|ccccccccccccc}
 & 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2 & \alpha \\
\hline
1 & -1 & -1 & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
S & & & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 \\
e_1 & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
e_2 & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
e_3 & & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
e_4 & & & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
e_5 & & & & & & & & \pm & \pm & \pm & \pm & \pm & \pm \\
e_6 & & & & & & & & & \pm & \pm & \pm & \pm & \pm \\
z_1 & & & & & & & & & & \pm & \pm & \pm & \pm \\
z_2 & & & & & & & & & & & \pm & \pm & \pm \\
b_1 & & & & & & & & & & & & \pm & \pm \\
b_2 & & & & & & & & & & & & -1 & \pm \\
\alpha & & & & & & & & & & & & &
\end{array}
$$

A priori 66 independent coefficients $\rightarrow$ 2^{66} distinct vacua

The twisted matter spectrum:

$$B^1_{\ell^1_3\ell^1_4\ell^1_5\ell^1_6} = S + b_1 + \ell^1_3 e_3 + \ell^1_4 e_4 + \ell^1_5 e_5 + \ell^1_6 e_6$$

$$B^2_{\ell^2_1\ell^2_2\ell^2_5\ell^2_6} = S + b_2 + \ell^2_1 e_1 + \ell^2_2 e_2 + \ell^2_5 e_5 + \ell^2_6 e_6$$

$$B^3_{\ell^3_1\ell^3_2\ell^3_3\ell^3_4} = S + b_3 + \ell^3_1 e_1 + \ell^3_2 e_2 + \ell^3_3 e_3 + \ell^3_4 e_4 \qquad l^j_i = 0,1$$

sectors $B^i_{pqrs} \quad \rightarrow \quad 16$ or $\overline{16}$ of $SO(10)$ with multiplicity $(1,0,-1)$

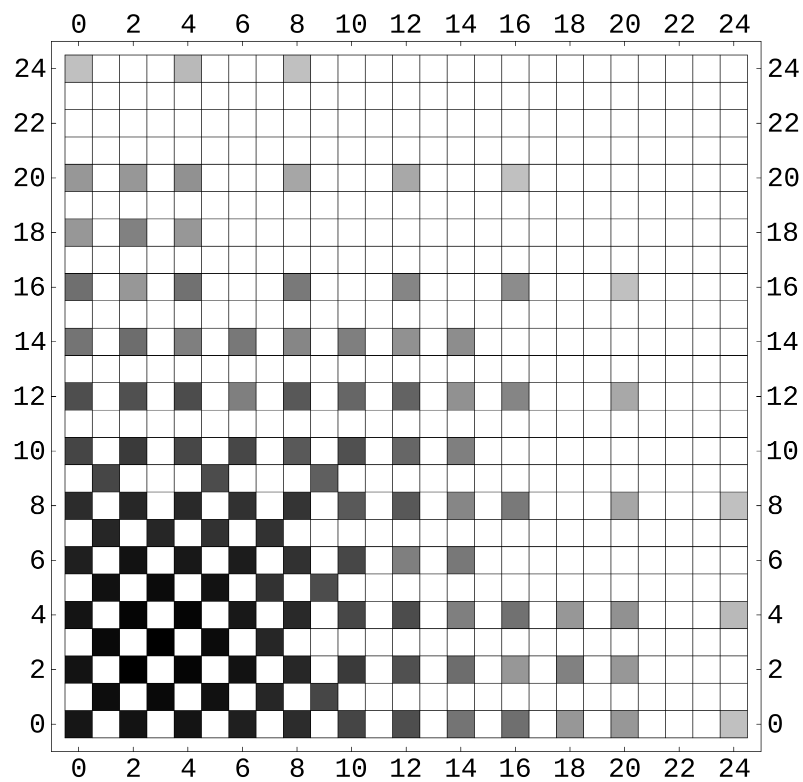
$B^i_{pqrs} + x \quad \rightarrow \quad 10$ of $SO(10)$ with multiplicity $(1,0)$

$x = \{\bar\psi^{1,\cdots,5}, \bar\eta^1, \bar\eta^2, \bar\eta^3\}$ $\qquad x-$ map $\leftrightarrow$ spinor–vector map

Algebraic formulas for $S = \sum_{i=1}^3 S^{(i)}_+ - S^{(i)}_-$ and $V = \sum_{i=1}^3 V^{(i)}$

Spinor–vector duality:

Invariance under exchange of $\#(16+\overline{16}) \;<->\; \#(10)$



Symmetric under exchange of rows and columns

$$E_6 \;:\qquad 27 = 16 + 10 + 1 \qquad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16+\overline{16}) \;=\; \#(10)$ without E_6 symmetry

Spinor–Vector duality in Orbifolds:

Using the level-one $\mathrm{SO}(2n)$ characters

$$O_{2n} = \tfrac{1}{2}\left(\frac{\theta_3^n}{\eta^n} + \frac{\theta_4^n}{\eta^n}\right), \qquad V_{2n} = \tfrac{1}{2}\left(\frac{\theta_3^n}{\eta^n} - \frac{\theta_4^n}{\eta^n}\right),$$

$$S_{2n} = \tfrac{1}{2}\left(\frac{\theta_2^n}{\eta^n} + i^{-n}\frac{\theta_1^n}{\eta^n}\right), \qquad C_{2n} = \tfrac{1}{2}\left(\frac{\theta_2^n}{\eta^n} - i^{-n}\frac{\theta_1^n}{\eta^n}\right).$$

where

$$\theta_3 \equiv Z_f\binom{0}{0} \quad \theta_4 \equiv Z_f\binom{0}{1}$$

$$\theta_2 \equiv Z_f\binom{1}{0} \quad \theta_1 \equiv Z_f\binom{1}{1}$$

Starting from:

$$Z_{+} = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} (\bar{O}_{16} + \bar{S}_{16}) \, (\bar{O}_{16} + \bar{S}_{16}) \ ,$$

where as usual, for each circle,

$$p^i_{\mathrm{L,R}} = \frac{m_i}{R_i} \pm \frac{n_i R_i}{\alpha'} \,,$$

and

$$\Lambda_{m,n} = \frac{q^{\frac{\alpha'}{4} p^2_{\mathrm{L}}} \, \bar{q}^{\frac{\alpha'}{4} p^2_{\mathrm{R}}}}{|\eta|^2} \,.$$

apply $\quad Z_2 \times Z_2' : g \times g'$

$$g \;:\; (-1)^{(F_1+F_2)}\delta$$

$$F_{1,2} \;:\; (\,\overline{O}_{16}^{1,2},\ \overline{V}_{16}^{1,2},\ \overline{S}_{16}^{1,2},\ \overline{C}_{16}^{1,2}\,) \longrightarrow (\,\overline{O}_{16}^{1,2},\ \overline{V}_{16}^{1,2},\ -\overline{S}_{16}^{1,2},\ -\overline{C}_{16}^{1,2}\,)$$

with $\quad \delta X_9 = X_9 + \pi R_9$,

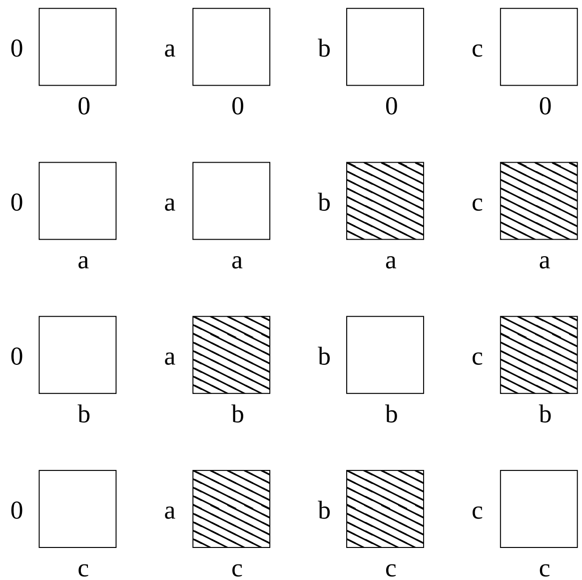
$$\delta \;:\; \Lambda^9_{m,n} \longrightarrow (-1)^m \Lambda^9_{m,n}$$

$$g' \;:\; (x_4, x_5, x_6, x_7, x_8, x_9) \longrightarrow (-x_4, -x_5, -x_6, -x_7, +x_8, +x_9)$$

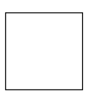
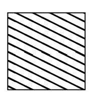
Note: A single space twisting Z_2' $\quad \Rightarrow \quad$ $N=4 \rightarrow N=2$

$$E_7 \rightarrow SO(12) \times SU(2)$$

$$\Rightarrow \quad \textsf{Analyze} \qquad Z = \left(\frac{Z_+}{Z_g \times Z_{g'}}\right) = \left[\frac{(1+g)}{2}\frac{(1+g')}{2}\right] Z_+$$



a = g ; b = g’ ; c = gg’

P.F. = ( + ε ) = $\Lambda_{m,n}$ • ( ) + $\Lambda_{m,n+1/2}$ • ( )

$\varepsilon = \pm 1$

massless massive

• sector b

$$\Lambda_{p,q}\left\{\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4+\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s \overline{V}_{12}\overline{C}_4\overline{O}_{16}+P_\epsilon^- Q_s \overline{S}_{12}\overline{O}_4\overline{O}_{16}\right]+\right.$$

$$\left.\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4-\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s \overline{O}_{12}\overline{S}_4\overline{O}_{16}\right]\right\} \quad + \quad \text{massive}$$

where

$$P_\epsilon^+ = \left(\frac{1+\epsilon(-1)^m}{2}\right)\Lambda_{m,n} \qquad P_\epsilon^- = \left(\frac{1-\epsilon(-1)^m}{2}\right)\Lambda_{m,n}$$

$$\epsilon = +1 \Rightarrow P_\epsilon^+ = \Lambda_{2m,n} \qquad P_\epsilon^- = \Lambda_{2m+1,n}$$

$$\epsilon = -1 \Rightarrow P_\epsilon^+ = \Lambda_{2m+1,n} \qquad P_\epsilon^- = \Lambda_{2m,n}$$

and $12\cdot 2 + 4\cdot 2 = 32$

Further :

- The spinor–vector duality in this model is realised in terms of a continuous interpolation between two discrete Wilson lines.

- The spinor-vector duality is realised in terms of a spectral flow operator that operates in the bosonic side of the heterotic string. In the case of enhanced E_6 symmetry, the spectral flow operator acts as an internal E_6 generator. When E_6 is broken the spectral flow operator induces the spinor–vector duality map.

Pati–Salam models statistics with respect to phenomenological constraints

constraint	# of models	probability	# of models
None	100000000000	1	2.25×10^{15}
+ No gauge group enhancements.	78977078333	7.90×10^{-1}	1.78×10^{15}
+ Complete families	22497003372	2.25×10^{-1}	5.07×10^{14}
+ 3 generations	298140621	2.98×10^{-3}	6.71×10^{12}
+ PS breaking Higgs	23694017	2.37×10^{-4}	5.34×10^{11}
+ SM breaking Higgs	19191088	1.92×10^{-4}	4.32×10^{11}
+ No massless exotics	121669	1.22×10^{-6}	2.74×10^{9}

Constraints in second column act additionally.

Exemplary Model (Christodoulides, AEF, Rizos, PLB 702 (2011) 81)

- A specific choice of one–loop GSO phases
- Analysis of cubic level superpotential and flat directions
- Only one Yukawa coupling at cubic level -> heavy family
- All extra colour triplets -> massive
- One light Higgs bi–doublet
- Solely MSSM below PS breaking scale

Away from the free fermionic point:

$$Z = \int \frac{d^2\tau}{\tau_2^2} \frac{\tau_2^{-1}}{\eta^{12}\bar\eta^{24}} \frac{1}{2^3} \left(\sum (-)^{a+b+ab} \vartheta \begin{bmatrix} a \\ b \end{bmatrix} \vartheta \begin{bmatrix} a+h_1 \\ b+g_1 \end{bmatrix} \vartheta \begin{bmatrix} a+h_2 \\ b+g_2 \end{bmatrix} \vartheta \begin{bmatrix} a+h_3 \\ b+g_3 \end{bmatrix} \right)_{\psi^\mu,}$$

$$\times \left(\frac{1}{2} \sum_{\epsilon,\xi} \bar\vartheta \begin{bmatrix} \epsilon \\ \xi \end{bmatrix}^5 \bar\vartheta \begin{bmatrix} \epsilon+h_1 \\ \xi+g_1 \end{bmatrix} \bar\vartheta \begin{bmatrix} \epsilon+h_2 \\ \xi+g_2 \end{bmatrix} \bar\vartheta \begin{bmatrix} \epsilon+h_3 \\ \xi+g_3 \end{bmatrix} \right)_{\bar\psi^{1\ldots5},\bar\eta^{1,2,3}}$$

$$\times \left(\frac{1}{2} \sum_{H_1,G_1} \frac{1}{2} \sum_{H_2,G_2} (-)^{H_1G_1+H_2G_2} \bar\vartheta \begin{bmatrix} \epsilon+H_1 \\ \xi+G_1 \end{bmatrix}^4 \bar\vartheta \begin{bmatrix} \epsilon+H_2 \\ \xi+G_2 \end{bmatrix}^4 \right)_{\bar\phi^{1\ldots8}}$$

$$\times \left(\sum_{s_i,t_i} \Gamma_{6,6} \begin{bmatrix} h_i | s_i \\ g_i | t_i \end{bmatrix} \right)_{(y\omega\bar y\bar\omega)^{1\ldots6}} \times e^{i\pi\Phi(\gamma,\delta,s_i,t_i,\epsilon,\xi,h_i,g_i,H_1,G_1,H_2,G_2)}$$

$$\Gamma_{1,1}\left[{}^h_g\right] = \frac{R}{\sqrt{\tau_2}} \sum_{\tilde m,n} \exp\left[-\frac{\pi R^2}{\tau_2} \left| (2\tilde m + g) + (2n+h)\,\tau \right|^2 \right]$$

Towards String Predictions

1. Low energy supersymmetry

   Specific SUSY breaking patterns $\longrightarrow$ Collider implications

2. Additional (non–GUT) gauge bosons

   Proton Stability and low–scale Z' $\longrightarrow$ Collider signatures

3. Exotic matter

   In realistic string models

   Unifying gauge group $\Rightarrow$ broken by "Wilson lines".

   $\Rightarrow$ non–GUT physical states.

   $\Rightarrow$ Meta-stable heavy string relics

   $\rightarrow$ Dark Mater ; UHECR candidates

Conclusions

- DATA $\longrightarrow$ UNIFICATION
- STRINGS $\longrightarrow$ GAUGE & GRAVITY UNIFICATION
- EXPERIMENTAL PREDICTIONS ?
- FUNDAMENTAL PRINCIPLES ?

  $e.g.$ spinor–vector duality $\rightarrow$ Physics & Geometry

  phase–space duality & the equivalence postulate of QM