Particle Emission from a Higher-Dimensional Black Hole

Panagiota Kanti

Department of Mathematical Sciences,
University of Durham

University of Liverpool, UK,
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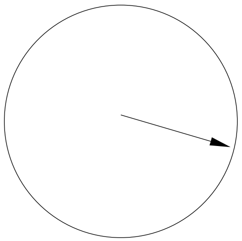
Outline :

1. Extra Dimensions and Black Holes
2. Evaporation of Black Holes - Hawking Radiation
   - Schwarzschild Black Holes (‘Schwarzschild phase’)
   - Kerr Black Holes (‘spin-down phase’)
   - Schwarzschild-de Sitter Black Holes
3. Conclusions

In collaboration with : J. March-Russell (Oxford),
S. Dolan & C. Harris (Cambridge), E. Winstanley (Sheffield),
G. Duffy & M. Casals (Dublin),
J. Grain & A. Barrau (Grenoble), S. Creek (Durham)

Introduction : Extra Dimensions

- Kaluza & Klein (1921/1926): Apart from the usual 3 spacelike dimensions, additional, compact spacelike dimensions may exist in nature



Why we can not observe them? Because of their tiny size $\mathcal{R}$: for experiments at $r \gg \mathcal{R}$, the extra dimensions are 'invisible'

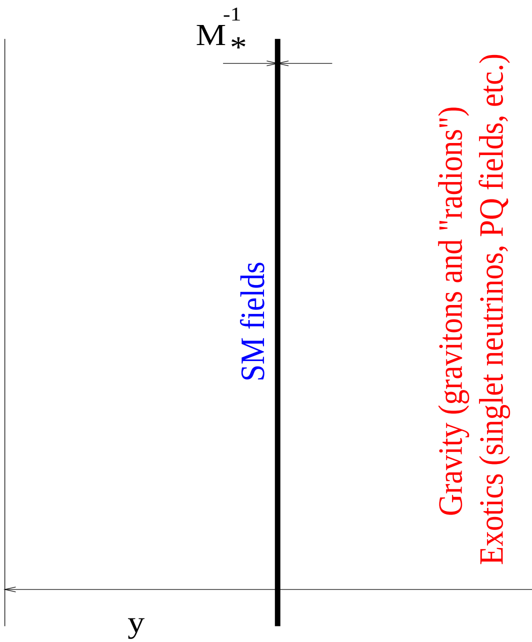
- Superstring Theory: Formulated in $D = 10$ dims
  - (1975-90): $\mathcal{R} = l_P = 10^{-33}$ cm
  - (90's): $\mathcal{R} \geq (TeV)^{-1}$

  (Antoniadis; Lykken; Horava & Witten)

Introduction : Extra Dimensions

- Large Extra Dimensions (1998) :

  - A 4D **Brane** with all the SM fields and scale for gravity $M_P = 10^{19}$ GeV
  - A $(4+n)$D Extra Space (**Bulk**) with gravitons and scale for gravity M_*
  - Then, we obtain:



$$\boxed{M_P^2 \simeq \mathcal{R}^n \, M_*^{2+n}}$$

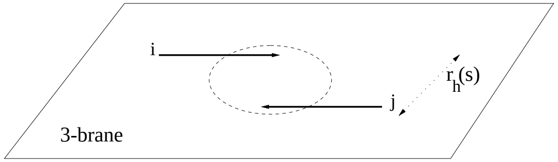
If $\mathcal{R} \leq 1$ mm, then $M_* \geq 1$ TeV $\Rightarrow$ resolution of the hierarchy problem (Arkani-Hamed, Dimopoulos & Dvali)

Introduction : Black Hole Creation

• During gravitational collapse on the brane, we expect the formation of a black hole, that will extend off the brane

• Small BH's with $r_H \ll \mathcal{R}$ are $(4+n)$-D objects

▷ Such Black Holes may be created during the scattering of high energy particles (Banks & Fischler; Giddings & Thomas; Dimopoulos & Landsberg)



For every center-of-mass-energy $\sqrt{s}$, there is a Schwarzschild radius r_H – if $b < r_H$, a black hole will be created (Thorne)

▷ For a black hole to be classical, it must have:

• in $4+n$ Dimensions: $M_{BH} > M_* \geq 1$ TeV

Introduction : Black Hole Creation

There are 2 ways to study the creation of those microscopic BHs:

- the geometrical method: the creation of a closed-trapped surface around the colliding particles/shock-waves (Penrose; Eardley & Giddings; Yoshino & Nambu; Kohlprath and Veneziano)

$$b_{\rm max} \sim 2^{-1/(D-3)}\, r_H \;\Longrightarrow\; \sigma_{\rm production} \sim \pi b_{\rm max}^2$$

Also:

$$M_{BH}(b,D) \geq (0.45 - 0.71)\,\sqrt{s}$$

As D increases, $\sigma_{\rm production}$ is enhanced but M_{BH} is suppressed – more but lighter black holes are created

- the perturbative method: calculation of the emitted gravitational radiation (D'Eath & Payne; Cardoso & Lemos; Berti, Cavaglia & Gualtieri)

$$E_{\rm grav} \sim (16\% - 8\%)\sqrt{s} \qquad \text{for} \qquad (D = 4 - 10)$$

Introduction : Black Hole Creation

- Realistic Collision: The colliding particles may be composite particles – we have to sum over all partons carrying enough energy to produce a microscopic, but classical, black hole. For example, for $M_* = 1$ TeV:

$M_{BH} = 5$ TeV	$M_{BH} = 10$ TeV
$\sigma_{\rm production} \sim 10^5$ fb	$\sigma_{\rm production} \sim 10^1$ fb

(Giddings & Thomas; Dimopoulos & Landsberg)

- An over-estimate? Yes, since it has been assumed that:

$$\sigma_{\rm production} \sim \pi r_H^2 \qquad \text{and} \qquad M_{BH} \simeq \sqrt{s}$$

Introduction : Black Hole Creation

• Stages of the life of the produced black hole: A highly asymmetric, rotating object that goes through the following: (Giddings & Thomas)

- Balding phase: shedding of all quantum numbers apart from (M, Q, A) – mainly invisible energy
- Spin-down phase: Loss of angular momentum – Hawking radiation – visible energy
- Schwarzschild phase: Loss of mass – Hawking radiation – visible energy
- Planck phase: when $M_{BH} \sim M_*$ – a few energetic quanta, or a stable "quantum" remnant?

The Schwarzschild Phase

- The gravitational background: A spherically-symmetric $(4+n)$-dimensional black hole with line-element

(Tangherlini; Myers & Perry)

$$ds^2 = -\left[1-\left(\frac{r_H}{r}\right)^{n+1}\right] dt^2 + \left[1-\left(\frac{r_H}{r}\right)^{n+1}\right]^{-1} dr^2 + r^2 d\Omega_{2+n}^2$$

where

$$d\Omega_{2+n}^2 = d\theta_{n+1}^2 + \sin^2\theta_{n+1}\Big(d\theta_n^2 + \sin^2\theta_n\Big(... + \sin^2\theta_2(d\theta_1^2 + \sin^2\theta_1 d\varphi^2$$

The above black hole has significantly modified properties compared to a similar 4-dimensional one

(Argyres, Dimopoulos & March-Russell)

The Schwarzschild Phase

▷ Horizon Radius:

$$\boxed{r_H = \frac{1}{M_*}\left(\frac{M}{M_*}\right)^{\frac{1}{n+1}}\left(\frac{8\Gamma(\frac{n+3}{2})}{(n+2)\sqrt{\pi}^{(n+1)}}\right)^{1/(n+1)}}$$

Horizon radii for $M_* = 1$ TeV and $M_{BH} = 5$ TeV

n	1	2	3	4	5	6	7
r_H (10^{-4} fm)	4.06	2.63	2.22	2.07	2.00	1.99	1.99

For $M_{BH} = 5$ TeV and $n = 0$, $r_H = 10^{-35}$ fm !!

The Schwarzschild Phase

▷ Temperature:

$$\boxed{T_H = \frac{(1+n)}{4\pi\, r_H}}$$

Temperatures for $M_* = 1$ TeV and $M_{BH} = 5$ TeV

n	1	2	3	4	5	6	7
T_{BH} (GeV)	77	179	282	379	470	553	629

- Typical lifetime: $\tau = (1.7 - 0.5) \times 10^{-26}$ sec for $n = 1 - 7$

The Schwarzschild Phase

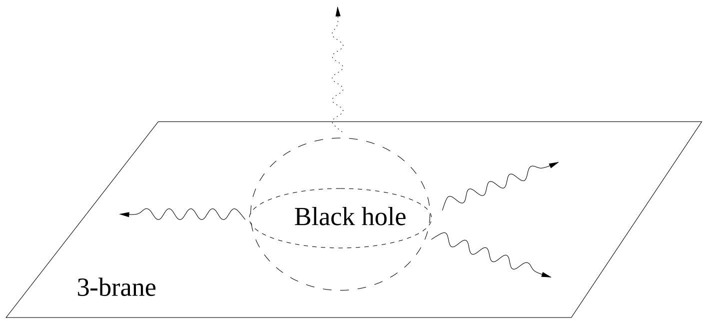
- Hawking Radiation: What is it?
  - creation of a virtual pair of particles just outside the horizon
  - the antiparticle falls into the BH whose mass decreases
  - the particle escapes to infinity where it gets observed

The Radiation Spectrum: A nearly black-body spectrum with emission rate

$$\boxed{\frac{dE(\omega)}{dt} = \frac{|\mathcal{A}_n^{(s)}(\omega)|^2\ \omega}{\exp\left(\omega/T_{BH}\right) \mp 1}\ \frac{d\omega}{(2\pi)}}$$

where $|\mathcal{A}_n^{(s)}(\omega)|^2$ is the absorption probability (greybody factor)

The Schwarzschild Phase



▷ Particles in the Bulk : Gravitons, Scalars

▷ Particles on the Brane : (zero-mode) Scalars, Fermions, Gauge Bosons, (zero-mode) Gravitons

The brane particles live in the projected $4D$ background ($\theta_n = \frac{\pi}{2}$)

$$ds^2 = -\left[1 - \left(\frac{r_H}{r}\right)^{n+1}\right] dt^2 + \left[1 - \left(\frac{r_H}{r}\right)^{n+1}\right]^{-1} dr^2 + r^2\, d\Omega_2^2$$

The Schwarzschild Phase

- Emission on the brane: To find the Absorption Probability, we must solve the corresponding Equation of Motion

(4D : Teukolsky & Press; Starobinsky & Churilov; Unruh; Sanchez; Page; McGibbon & Webber)

$$\boxed{\Delta^s \frac{d}{dr}\left(\Delta^{1-s}\frac{dP_s}{dr}\right) + \left[\frac{\omega^2 r^2}{h} + 2i\omega s r - \frac{is\omega r^2 h'}{h} - \Lambda_{sj}\right] P_s(r) = 0}$$

(P.K.)

where

$$\Psi_s = e^{-i\omega t}\, e^{im\varphi}\, \Delta^{-s}\, P_s(r)\, S^m_{s,j}(\theta)$$

$$\Delta = r^2\, h = r^2 \left[1 - \left(\frac{r_H}{r}\right)^{n+1}\right], \qquad \Lambda_{sj} = j(j+1) - s(s-1)$$

The Schwarzschild Phase

This equation may be solved....

- Analytically : An approximation method must be followed :

(P.K. & March-Russell)

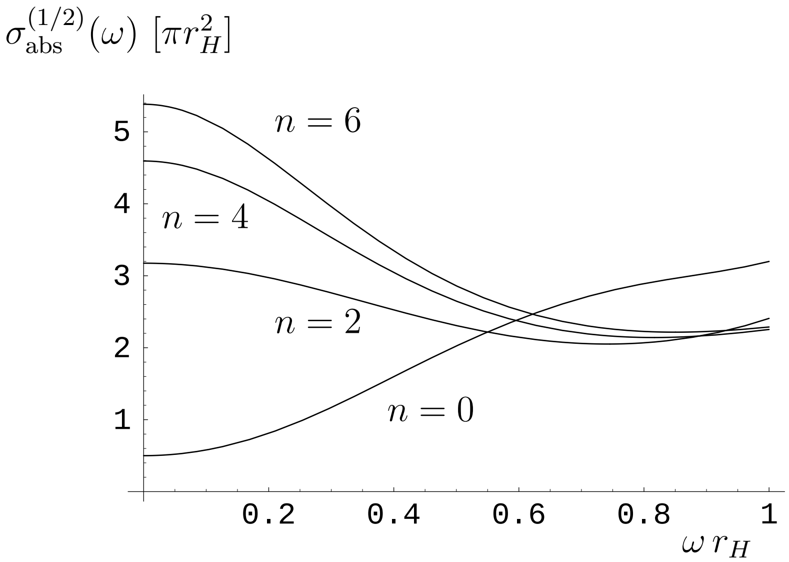
  - find the solution in the Near-Horizon regime $(r \simeq r_H)$
    – a solution of a hypergeometric equation
  - find the solution in the Far-Field regime $(r \gg r_H)$
    – the solution of a confluent hypergeometric equation
  - match the two solutions in an intermediate zone

Once $P_s(r)$ is found, we compute the Absorption Probability:

$$\boxed{|\mathcal{A}^{(s)}_{n,\ell}(\omega)|^2 = \frac{\mathcal{F}_{\rm horizon}}{\mathcal{F}_{\rm infinity}}}$$

The Schwarzschild Phase

- E.g. for fermions, we find that $\sigma_\ell(\omega) = \frac{\pi}{\omega^2}\,(2\ell+1)\,|\mathcal{A}^{(s)}_{n,\ell}(\omega)|^2$ has the behaviour



The analytic result is valid only at the low-energy regime - during the matching we assumed that $\omega r_H \ll 1$

The Schwarzschild Phase

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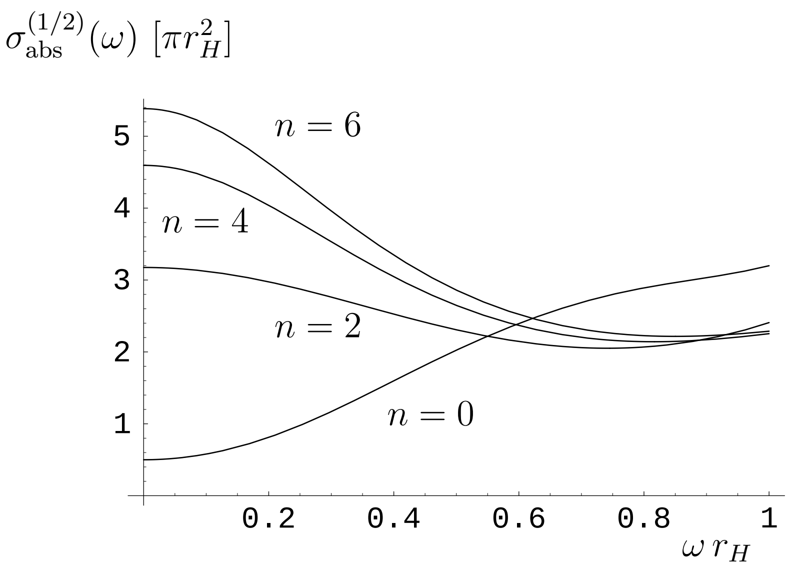

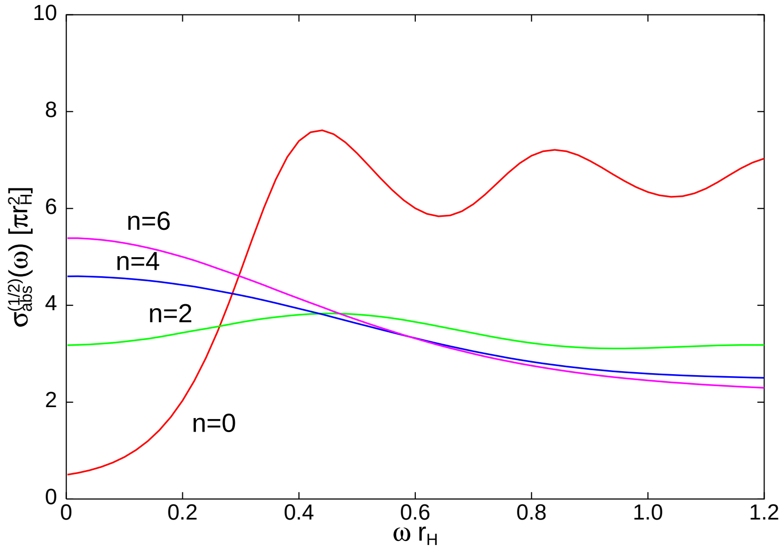

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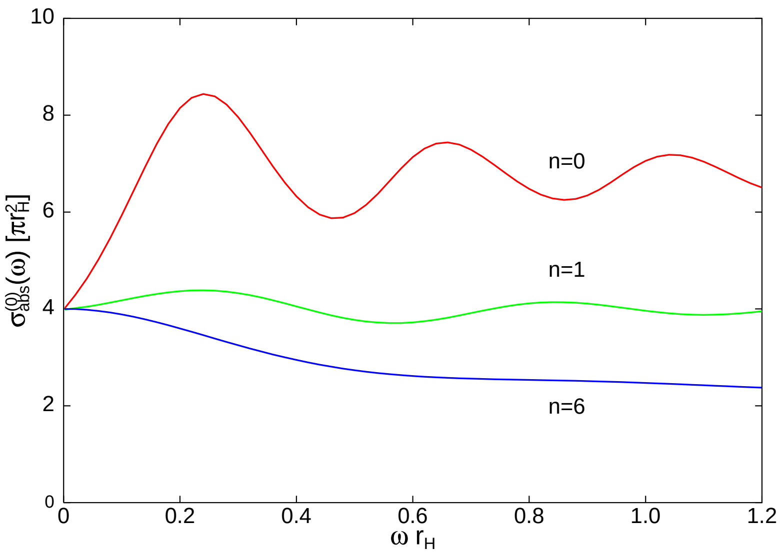
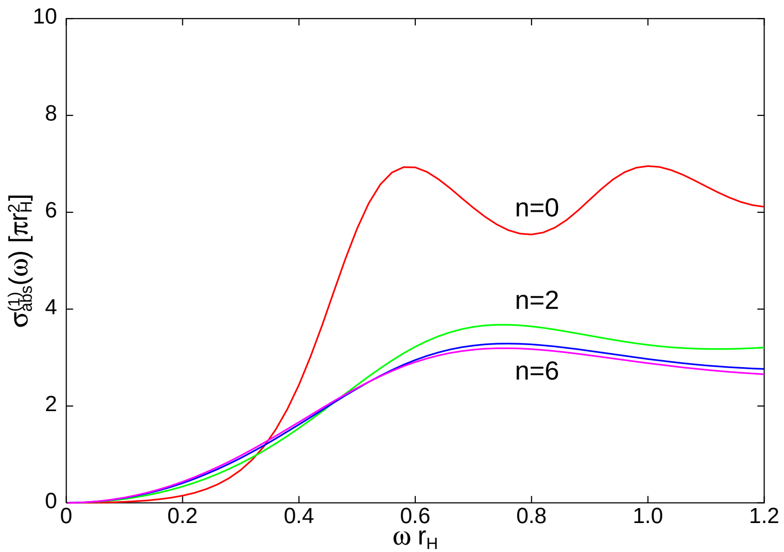
- Numerically: For results valid at all energy regimes, we have to use numerical analysis (Harris & P. K.)

The Schwarzschild Phase

- Similarly, for scalars and gauge bosons, we find respectively:

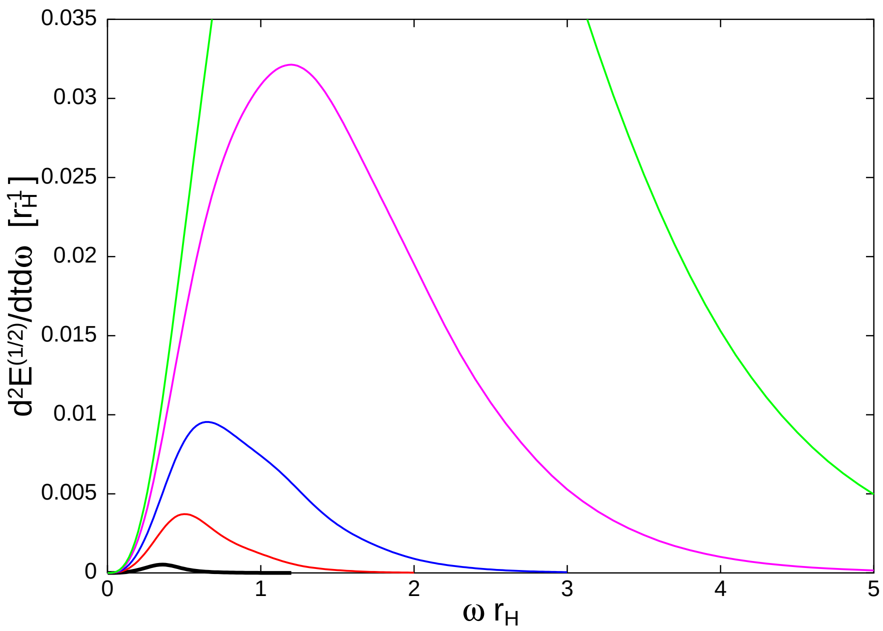




(Harris & P. K.)

The Schwarzschild Phase

- The amount of energy emitted per unit time strongly depends on the number of transverse-to-the-brane spacelike dimensions

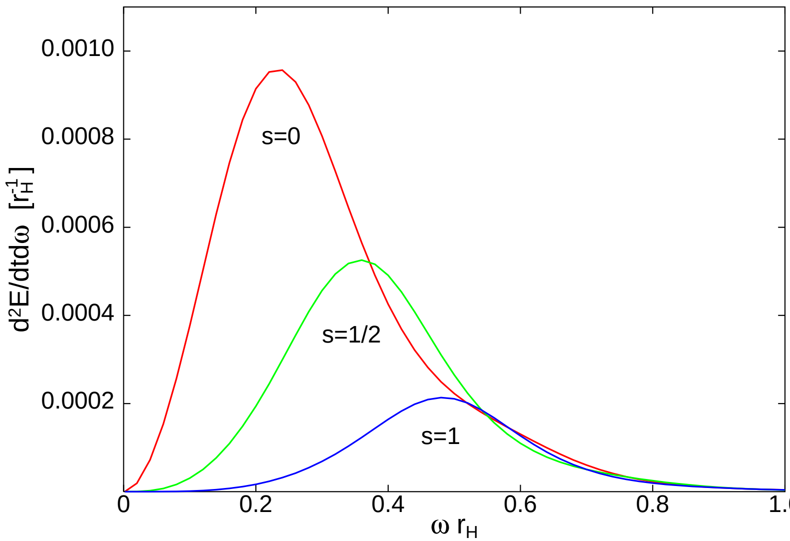


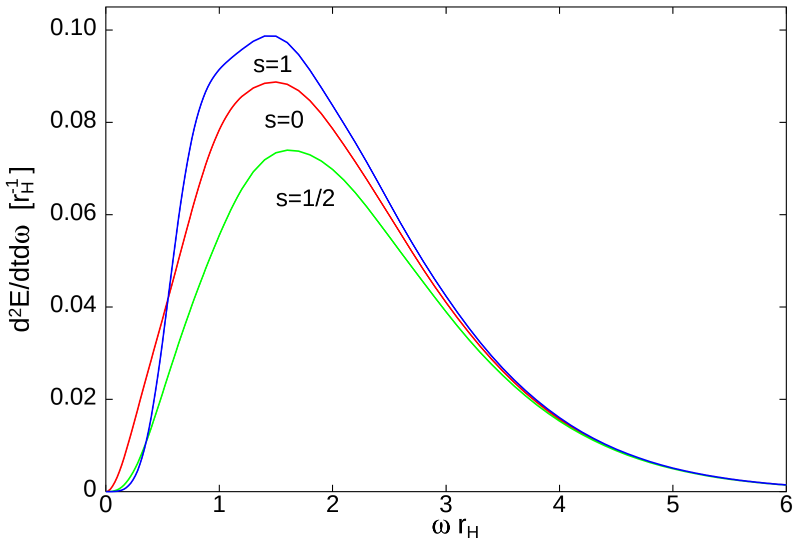
n	0	1	2	3	4	5	6	7
Scalars	1.0	8.94	36.0	99.8	222	429	749	1220
Fermions	1.0	14.2	59.5	162	352	664	1140	1830
G. Bosons	1.0	27.1	144	441	1020	2000	3530	5740

The Schwarzschild Phase

- Relative Emission Rates : How do they change with n?

(Harris & P. K.)





$(n = 0)$ $\boxed{1 : 0.55 : 0.23}$ $(n = 6)$ $\boxed{1 : 0.84 : 1.06}$

- The type of the emitted radiation also depends strongly on n

The Spin-down (Kerr) Phase

- The line-element on the brane has the form of a rotating, neutral, n-dependent Myers-Perry solution

$$ds^2 = \left(1 - \frac{\mu}{\Sigma\, r^{n-1}}\right) dt^2 + \frac{2a\mu \sin^2\theta}{\Sigma\, r^{n-1}}\, dt\, d\varphi - \frac{\Sigma}{\Delta} dr^2$$

$$-\Sigma\, d\theta^2 - \left(r^2 + a^2 + \frac{a^2\mu \sin^2\theta}{\Sigma\, r^{n-1}}\right) \sin^2\theta\, d\varphi^2,$$

where

$$\Delta = r^2 + a^2 - \frac{\mu}{r^{n-1}} \qquad \text{and} \qquad \Sigma = r^2 + a^2 \cos^2\theta$$

and the parameters μ and a are associated to the black hole mass and angular momentum as

$$M = \frac{(n+2)A_{n+2}}{16\pi G}\,\mu \qquad \text{and} \qquad J = \frac{2}{n+2}\, M a$$

The Spin-down (Kerr) Phase

- In this case, the "master" e.o.m. becomes (P.K.)

$$\Delta^{-s}\,\frac{d}{dr}\left(\Delta^{s+1}\,\frac{dR_s}{dr}\right)+$$

$$\left[\frac{K^2-iKs\Delta'}{\Delta}+4is\omega r+s\,(\Delta''-2)-\Lambda_\ell^m\right]R_s=0$$

where $K=(r^2+a^2)\,\omega-am$.

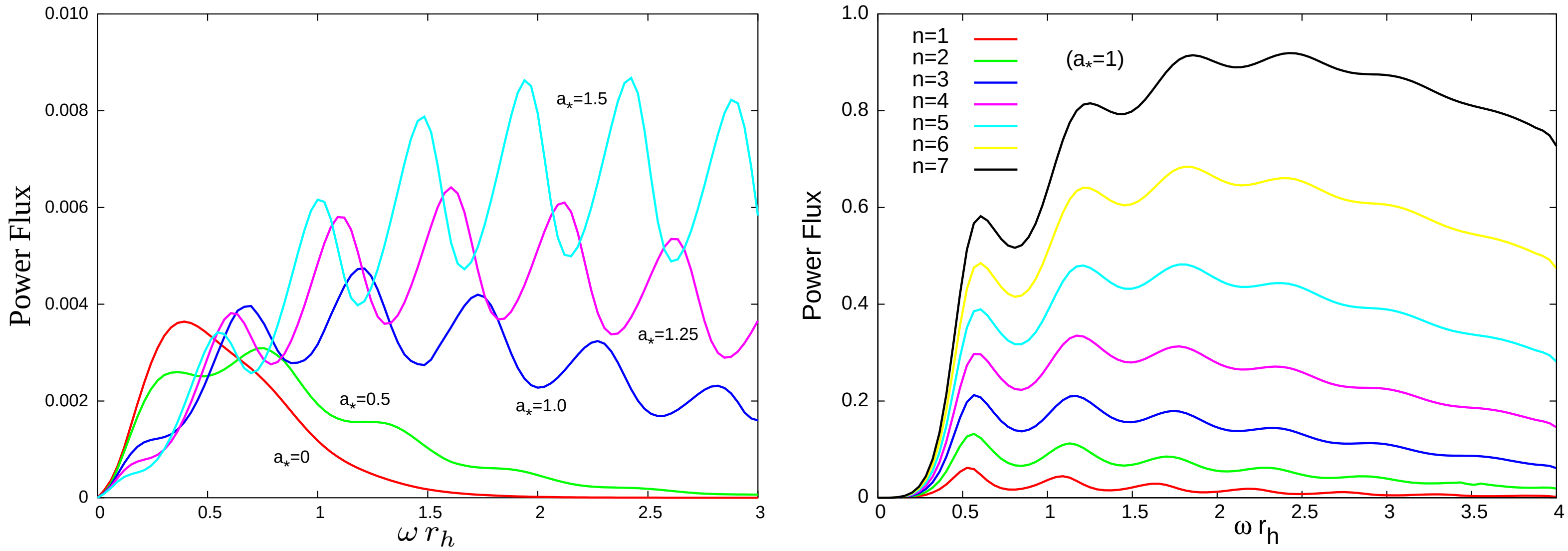
The Hawking temperature and rotation velocity of this brane black hole is

$$T_H=\frac{(n+1)\,r_H^2+(n-1)\,a^2}{4\pi(r_H^2+a^2)\,r_H}\,,\qquad \Omega=\frac{a}{(r_H^2+a^2)}$$

The Spin-down (Kerr) Phase

The differential energy emission rate is now given by:

$$\frac{d^2E(\omega)}{dt\,d\omega} = \sum_{\ell,m} |\mathcal{A}_{\ell,n}^m|^2 \,\frac{\omega}{\exp\left[(\omega - m\Omega)/T_H\right] - 1}\,\frac{1}{2\pi}$$



For all species of fields, as a or n increases, the energy emission rate is enhanced (P.K. and Harris; Duffy, Harris, P.K. & Winstanley; Casals, P.K. & Winstanley; Casals, Dolan, P.K. & Winstanley; Ida, Oda & Park)

The Spin-down (Kerr) Phase

- Relative Emissivities : An upper cut-off was imposed on the energy parameter ωr_H (3, 4 or 6) to reduce computing time. However, one may still deduce that the Gauge Bosons are the modes preferrably emitted by a rotating black hole

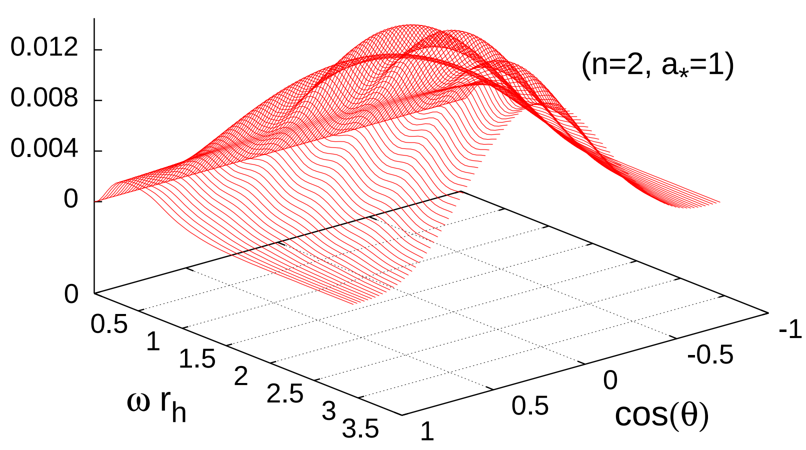
- Energy Emissivity : Enhancement Factors

	$(n=4)\ a=0$	$a=1.0$	$(a=1)\ n=1$	$n=7$
Scalars	1	≥ 3	1	≥ 100
Fermions	1	≥ 4	1	≥ 80
G. Bosons	1	≥ 5	1	≥ 50

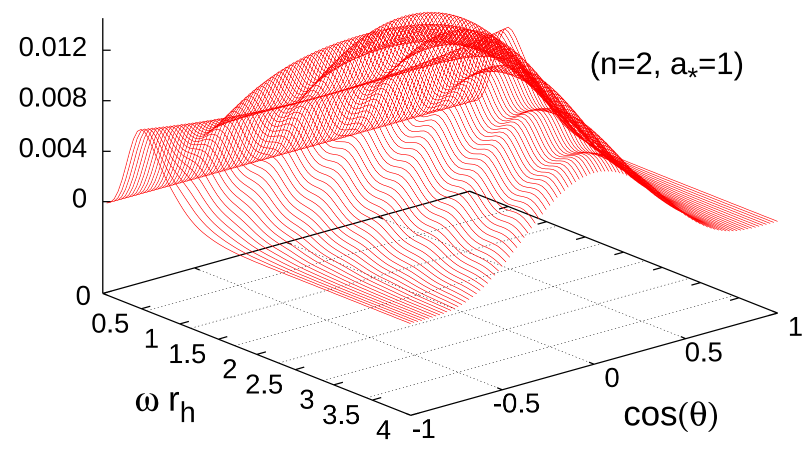
The Spin-down (Kerr) Phase

- Angular Distribution of the emitted power : For the different species of fields, $a = 1$ and $n = 2$, we find:

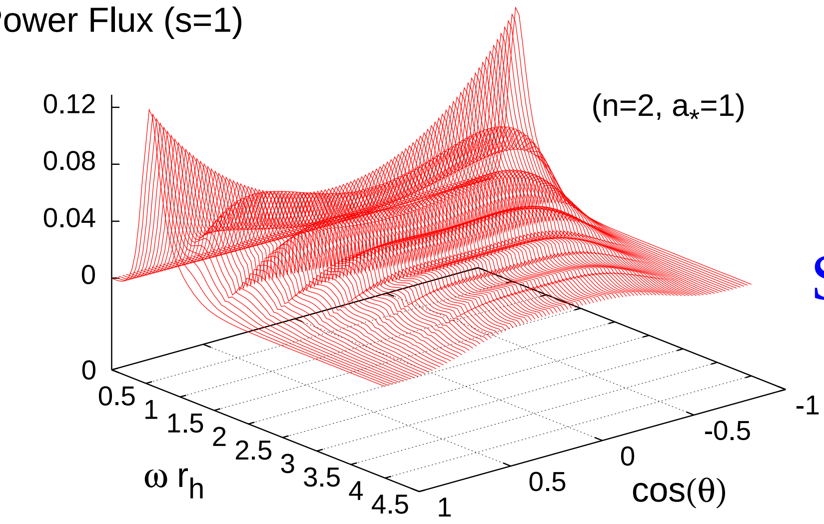
Power Flux (s=0)



Power Flux (s=1/2)



Power Flux (s=1)

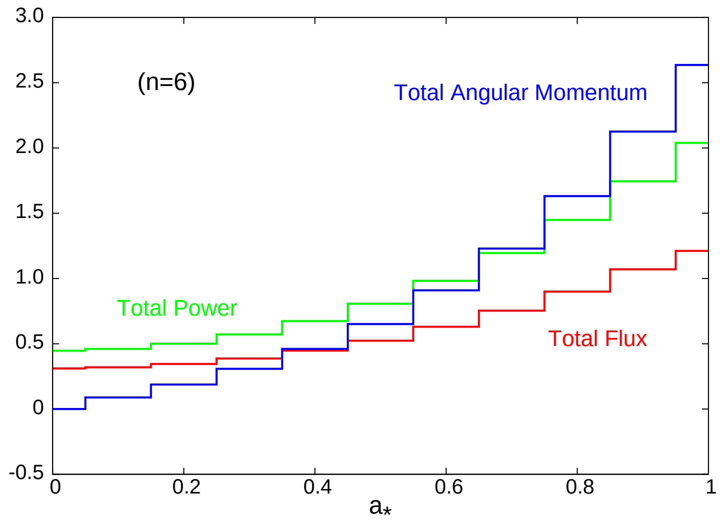


Centrifugal potential : emission on the equatorial plane

Spin-rotation coupling : emission parallel to the axis of rotation

The Spin-down (Kerr) Phase

- Evolution in time : For a realistic description, our results will be entered into the CHARYBDIS Code. Qualitatively, we observe:

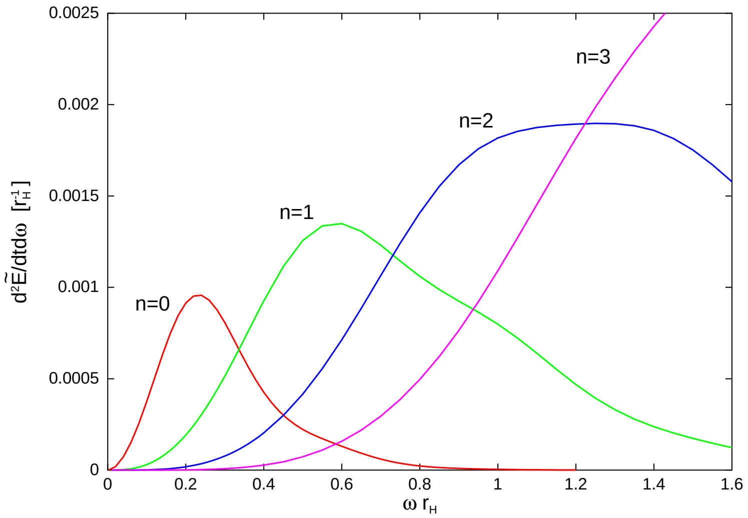


The Spin-down Phase is probably longer and more important, from the energetic point of view, than originally thought

(Ida, Oda & Park)

The Schwarzschild Phase

- Emission in the Bulk : For scalar fields emitted in the bulk, we find: (Harris & P. K.)



- Bulk-to-Brane Relative Emissivity : Where is most of the energy of the “scalar channel" emitted?

	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
Bulk/Brane	1.0	0.40	0.24	0.22	0.24	0.33	0.52	0.93

The Schwarzschild Phase

• Emission in the Bulk : The emission of gravitons was recently studied (Cardoso, Cavaglia & Gualtieri; Creek, Efthimiou, P.K. & Tamvakis; Park)

Tensor, Vector and Scalar gravitational modes are sub-dominant to the bulk scalar fields in the low-energy regime, but the relation is reversed in the high-energy regime.

• Bulk or Brane? Where is most of the energy of the “graviton channel" emitted? No results are yet available

No results for the emission spectra for scalars and gravitons have been derived for the spin-down phase

Schwarzschild - de Sitter Black Holes

- In the presence of a positive Cosmological Constant Λ, the geometrical background on the brane becomes:

$$ds^2 = -h(r)\,dt^2 + \frac{dr^2}{h(r)} + r^2 d\Omega_2^2$$

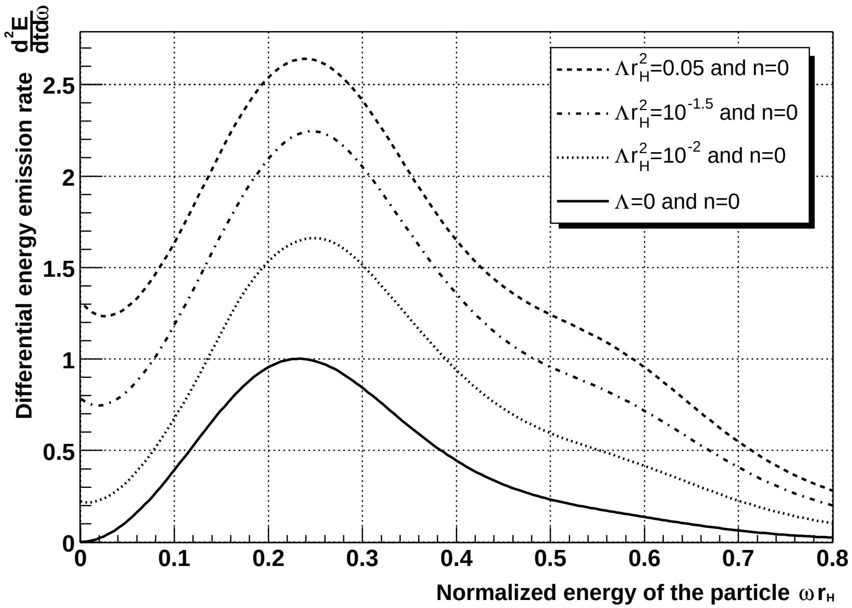
where

$$h(r) = 1 - \frac{\mu}{r^{n+1}} - \frac{2\kappa_D^2\,\Lambda\,r^2}{(n+3)(n+2)}$$

The equation $h(r) = 0$ has two real, positive solutions: r_H and r_C. The temperature of the BH is given by:

$$T_H = \frac{1}{\sqrt{h(r_0)}}\,\frac{1}{4\pi r_H}\left[\,(n+1) - \frac{2\kappa_D^2\Lambda}{(n+2)}\,r_H^2\,\right]$$

Schwarzschild - de Sitter Black Holes



(P.K., Grain & Barrau)

The energy emission rate is clearly enhanced with Λ

Moreover, there is a constant rate of emission of low-energy modes, that appears for any value of n, including $n = 0$

$$|\mathcal{A}_{0,n}(\omega = 0)|^2 = \frac{4r_C^2 r_H^2}{(r_C^2 + r_H^2)^2} \neq 0 \qquad (\to 0\,, \;\text{when} \quad r_C \to \infty)$$

Conclusions

- Until today, we have not yet detected the Hawking radiation from a decaying black hole

- If Extra Dimensions exist, then
  - the production of small black holes becomes possible
  - the detection of Hawking radiation becomes more likely

- The emission spectra can help us determine the dimensionality of spacetime with possible signatures being the rate and type of the emitted radiation

- The emission spectra will depend on additional parameters of the black-hole background, such as the angular momentum

- The spectrum will also depend on the cosmological constant of spacetime on which valuable information could also be extracted.

Introduction : Extra Dimensions

Current limits on the fundamental energy scale

Type of Experiment/Analysis	$M_* \geq$	$M_* \geq$
Collider limits on the production of real or virtual KK gravitons	1.45 TeV ($n = 2$)	0.6 TeV ($n = 6$)
Torsion-balance Experiments	3.5 TeV ($n = 2$)	
Overclosure of the Universe	8 TeV ($n = 2$)	
Supernovae cooling rate	30 TeV ($n = 2$)	2.5 TeV ($n = 3$)
Non-thermal production of KK modes	35 TeV ($n = 2$)	3 TeV ($n = 6$)
Diffuse gamma-ray background	110 TeV ($n = 2$)	5 TeV ($n = 3$)
Thermal production of KK modes	167 TeV ($n = 2$)	1.5 TeV ($n = 5$)
Neutron star core halo	500 TeV ($n = 2$)	30 TeV ($n = 3$)
Neutron star surface temperature	1700 TeV ($n = 2$)	60 TeV ($n = 3$)
BH absence in neutrino cosmic rays		1-1.4 TeV ($n \geq 5$)