Towards Spinor-Vector duality in smooth heterotic compactifications

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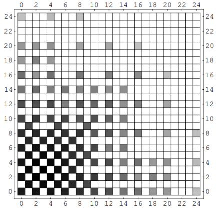
Index

- The classication of heterotic $N = 1$ $Z_2 \times Z_2$ models revealed a symmetry in the distribution of the vacua under exchange of vectorial, and spinorial plus antispinorial representations of $SO(10)$



- This symmetry was later observed in $N = 2$ fermionic Z_2 models. In this case the duality is between the interchange of spinor and vectorial (plus singets) of $SO(12)$

  SV duality: $\mathbf{32} \leftrightarrow \mathbf{12} \oplus \mathbf{1}$ such that the number of massless states are constant between two dual models

- The duality has been also studied in the context of SCFT theories $\rightarrow$ Spectral-flow operator in the twisted sector

- SV duality can be achieved by turning **Wilson lines** with certain values around non contractible loops of the orbifold $\rightarrow$ breaks the gauge group of the theory giving mass either to the spinor or vector representations.
- **Parallelism with mirror symmetry?** Mirror symmetry in the type II string says that CYs come in pairs X and Y such that $h^{1,1}(X) = h^{2,1}(Y)$ and $h^{2,1}(X) = h^{1,1}(Y)$ and consequently exchanges spinor and antispinor representations in the heterotic string ($\mathbf{16} \leftrightarrow \overline{\mathbf{16}}$ of $SO(10)$ for example)
  - This implies topology-changing transitions.

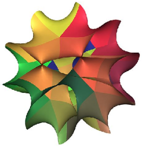
Question

How the SV duality manifests in the smooth limit $\rightarrow$ CY3s with vector bundles?

- We need to find the parameter (as certain values of Wilson lines in the orbifold case) controlling the duality in the smooth case.

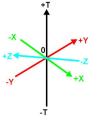
Basics of heterotic smooth compactifications

N=1



CY3

Π



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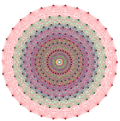
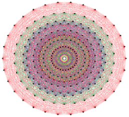


X



E8

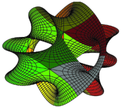
E8'

Usualy "hidden bundle"

Vector bundle with
structure group G
It contains the
gauge degrees of
freedom

The 4D gauge group will
be the commutant of H
in E8xE8 (generally in
the first E8)

N=2



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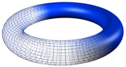


K3

T^2

$G \times H$	Breaking Pattern: $\mathbf{248} \rightarrow$	Particle Spectrum
$\mathrm{SU}(3) \times \mathrm{E}_6$	$(\mathbf{1}, \mathbf{78}) \oplus (\mathbf{3}, \mathbf{27}) \oplus (\overline{\mathbf{3}}, \overline{\mathbf{27}}) \oplus (\mathbf{8}, \mathbf{1})$	$n_{27} = h^1(V)$ $n_{\overline{27}} = h^1(V^\star) = h^2(V)$ $n_1 = h^1(V \otimes V^\star)$
$\mathrm{SU}(4) \times \mathrm{SO}(10)$	$(\mathbf{1}, \mathbf{45}) \oplus (\mathbf{4}, \mathbf{16}) \oplus (\overline{\mathbf{4}}, \overline{\mathbf{16}}) \oplus (\mathbf{6}, \mathbf{10}) \oplus (\mathbf{15}, \mathbf{1})$	$n_{16} = h^1(V)$ $n_{\overline{16}} = h^1(V^\star) = h^2(V)$ $n_{10} = h^1(\wedge^2 V)$ $n_1 = h^1(V \otimes V^\star)$
$\mathrm{SU}(5) \times \mathrm{SU}(5)$	$(\mathbf{1}, \mathbf{24}) \oplus (\mathbf{5}, \mathbf{10}) \oplus (\overline{\mathbf{5}}, \overline{\mathbf{10}}) \oplus (\mathbf{10}, \overline{\mathbf{5}}) \oplus (\overline{\mathbf{10}}, \mathbf{5}) \oplus (\mathbf{24}, \mathbf{1})$	$n_{10} = h^1(V)$ $n_{\overline{10}} = h^1(V^\star) = h^2(V)$ $n_5 = h^1(\wedge^2 V^\star)$ $n_{\overline{5}} = h^1(\wedge^2 V)$ $n_1 = h^1(V \otimes V^\star)$

↓

+ Wilson lines

↓

SM group

If the gauge connection is equal to the spin connecton of the manifold we will have an SU(3) (SU(2) in the N=2 case) vector bundle, which is unique.
For a larger G we have freedom and we require certain techniques to construct the bundle

Basics of heterotic smooth compactifications

In summary our ingredients are a Calabi-Yau threefold X, and two holomorphic vector bundles V and V' (holomorphicity is guaranteed if $F_{ab} = F_{\bar{a}\bar{b}}$) with structure group $SU(n)$ with $n = 2, 3, 4, 5$. They are subject to a certain physical **constraints**

Physical constraints

- **Bundle stability**

$$g^{a\bar{b}} F_{a\bar{b}} = 0 \underset{DUY}{\rightarrow} \mu(\mathcal{F}) < \mu(V) \ \ \forall \mathcal{F} \subset V \tag{1}$$

$$\left(\mu(V) = \frac{1}{rank(V)} \int_X c_1(V) \wedge \omega \wedge \omega \right) \rightarrow \text{"model dependent"} \tag{2}$$

- **Anomaly cancellation**

$$c_2(TX) - c2(V) - c2(V') = W_5 \tag{3}$$

- **Three generations**

$$\frac{1}{3} \left(\frac{1}{2} \int_X c_3(V) \right) = n\chi(X); n \in \mathbb{N} \tag{4}$$

As we said we can break further the gauge group by turning Wilson lines. To do so we must have a non-simply connected Calabi Yau.

$$X \to \frac{X}{\Gamma}; \text{ with } \Gamma = \prod_{N^i} \mathbb{Z}_{N^i} = \text{Wilson line group} \tag{5}$$

Methods for construction of vector bundles

One of the most involved pars of the heterotic model building is the determination of a vector bundle fulfilling the constraints described above. There are different methods.

Spectral cover

For elliptically CY3 $X \xrightarrow{\pi} B$ vector bundles with $G = SU(n)$ can be constructed in terms of spectral data:

- Spectral cover $\mathcal{C} \subset X$, which is a divisor in X of degree n
- A line bundle $\mathcal{N}$ on $\mathcal{C}$

The pair $(\mathcal{C}, \mathcal{N})$ can be written* in terms of an effective class $\eta \in H^2(B, \mathbb{Z})$ and coefficients λ κ_i which specify the bundle and fulfil

$$c_1(\mathcal{N}) = n\left(\frac{1}{2}+\lambda\right)\sigma + \left(\frac{1}{2}+\lambda\right)\pi^*\eta + \left(\frac{1}{2}+n\lambda\right)\pi^*\left(c_1(B)\right) \rightarrow \tag{6}$$

$$c_1(\mathcal{N}) \text{ is an integer class } \leftrightarrow \lambda \in \mathbb{Z}; \eta = c_1(B) \text{ mod} 2 \tag{7}$$

Monad bundles

For CYs defined as (intersection of) hypersurfaces in $\mathbb{P}^n$ (or in a product $\mathbb{P}^{n_1} \times ... \times \mathbb{P}^{n_r}$, i.e. CICYs) we can use the monad construction

$$0 \to V \to B \xrightarrow{f} C \to 0 \tag{8}$$

$$B = \bigoplus_{i=1}^{r_B} \mathcal{O}_X(\mathbf{b}_i),\ C = \bigoplus_{i=1}^{r_C} \mathcal{O}_X(\mathbf{c}_j) \tag{9}$$

The map f is given by a matrix of homogeneous polynomials of (multi)degrees $\mathbf{c}_j - \mathbf{b}_i$ (with non-trivial entries $(c_j^r \geq b_i^r \forall j, i, r)$) and determines the bundle as $V = ker(f)$ Furthermore we shall impose some more constraints:

$$rank(V) \stackrel{!}{=} 3, 4, 5;\ c_1^r(V) = \sum_{i=1}^{r_B} b_i^r - \sum_{j=1}^{r_C} c_j^r \forall r\ (SU(n),\ n = 3, 4, 5 \text{ bundles}) \tag{10}$$

$$c_{2r}(V) = \frac{1}{2} \kappa_{rst} \left(\sum_{j=1}^{r_C} c_j^s c_j^t - \sum_{i=1}^{r_B} b_i^s b_i^t \right) \leq c_{2r}(TX) \forall r \text{ Anomaly cancellation} \tag{11}$$

Methods for construction of vector bundles

Monad bundles

Stability is usually more involved and require using other exact sequences (Koszul and Leray exact sequences)

The particle spectrum will be determined by different cohomology groups of the bundle and can be worked out by cohomological methods (exact sequences, (Mcauley2,etc))

Divisors, and toric geometry

The method I have used more intensively, will be illustrated in the following sections,

A first step in $N = 1$: Schoen manifold

- A natural first step is the observation that some free fermionic $\mathcal{N} = 1$ models have realization as $\frac{T^6}{\mathbb{Z}_2 \times \mathbb{Z}_2}$ orbifolds which can be resolved into a smooth CY3, known as the **Schoen manifold**

- This manifold can be seen as a fibred product $B_1 \times_{\mathbb{P}^1} B_2$, as a CICY of the form $\left[\begin{array}{c||cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^2 & 0 & 3 \\ \mathbb{P}^2 & 3 & 0 \end{array}\right]^{19,19}$ for (monad bundles method) or as a resolution of the cited orbifold (for **Toric geometry and divisors method**).

Main idea of the toric geometric method

We define a set of divisors (hypersurfaces) $\{D_i\}$ in our manifold (both exceptional (coming for the resolution of the singularity) and ordinary ones) and, as they are characterized by $(1, 1)$, use them to expand the gauge backgroud $\mathcal{F}$:

$$\mathcal{F} = 2\pi \hat{D}_i H_i = 2\pi \hat{D}_i V_i^I H_I \tag{12}$$

with $i = 1, ..., h^{1,1}$=divisors and I=1,...,16 (H_I are the Cartan generators of the $E_8 \times E_8$ algebra

Following the previous equation we can see that each constant 16−component line bundle vector V_i characterizes how the line bundle, supported on the divisor D_i, is embedded in the Cartan subalgebra of the 10D gauge group $E_8 \times E_8$

Then the equations of this model will be:

$$\int_{C_i} \frac{\mathcal{F}(p))}{2\pi} \in \mathbb{Z} \to V = (V' \mid V'') \in (\Lambda_{G'} \mid \Lambda_{G''}) \text{ is a} \tag{13}$$

$(h^{1,1}, rank(G') + rank(G''))$-matrix of integers or half-integers

$$\text{Anomaly cancellation} \to \kappa_{ijk} \left(V_j^{'} V_k^{'} - V_j^{''} V_k^{''} \right) = 0 \tag{14}$$

$$\text{Stability (DUY equations)} : \frac{1}{2} \int_X \omega^2 \frac{\mathcal{F}}{2\pi} = Vol(D_i) V_i^I = 0 \leftrightarrow$$

$$\exists a_i, b_j > 0 / \left(\sum_{i=1}^{h^{1,1}} a_i V_i', \sum_{i=1}^{h^{1,1}} b_j V_j'' \right) = (\mathbf{0}, \mathbf{0}) \tag{15}$$

A first step in $N = 1$: Schoen manifold

Having defined tthe gauge background the 4D gauge group will be given by those roots $\{p \in \Lambda\}$ that are uncharged under $\mathcal{F} \leftrightarrow V_i \cdot p = 0 \forall i = 1, ..., h^{1,1}$

With these equations in hand we can build quite a large number of $SU(4)$ bundles over the Schoen manifold and calculate their spectra using the multiplicity operator for chiral matter

$$\mathcal{N}(p) = \frac{1}{6}\kappa_{ijk}(V_k p)(V_i p)(V_k p) + \frac{1}{12} c_{2i}(V_i p) \tag{16}$$

And the *orbifolder* package* for the Higgs part of the spectrum

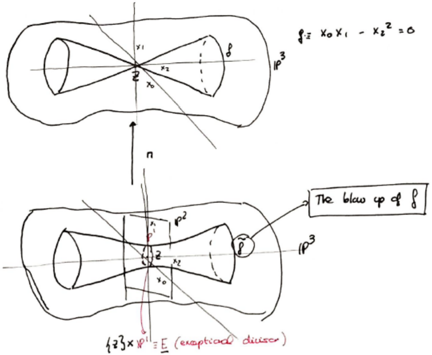
The calculations are ongoing and we are looking for different gauge backgrounds that give rise to SV dual models. Also we can make use of Wilson lines (to reduce the spectrum as in the orbifold case) taking into account that:

$$p \cdot W = 0 \bmod 1 \tag{17}$$

$$NW \in \Lambda \text{ with } \mathbb{Z}_N\text{-Wilson line} \tag{18}$$

In the $N = 2$ side we have a well studied model in which the SV duality is explicitly given in terms of certain values of the Wilson lines

This model is a $\frac{T^4}{\mathbb{Z}_2} \times T^2$ orbifold which can be resolved into a model that is a **K3** $\times$ $\mathbf{T^2}$ **with standard embedding (i.e. SU(2) bundle)**



A first step in $N = 2$: $K3 \times T^2$

- In the orbifold point of the moduli space we have an enhanced symmetry $G = E_7 \times SU(2) \times E_8' \times U(1)^4$

  Then a Wilson line can be turned along one of the S^1 of the T^2 to break the E_7 into $SO(12) \times SU(2)$ giving mass to the spinorial or vectorial (and singlet) representations of $SO(12)$

- In the smooth compactification we have (in a generic point of the moduli space) a gauge group $G = E_7 \times E_8' \times U(1)^4$ and the curvature of the $SU(2)$ bundle is concentrated in the 24 instantons (for anomaly cancellation) which are located in the same E_8 bundle

  The idea is to find some parameter which control the deformations of the $SU(2)$ vector bundle in an analogous way to the Wilson lines in the orbifold case

- Conformal Aspects of Spinor-Vector Duality; Alon E. Faraggi, Ioannis Florakis, Thomas Mohaupt, Mirian Tsulaia
- (MS)SM-like models on smooth Calabi-Yau manifolds from all three heterotic string theories; Stefan Groot Nibbelink. Orestis Loukas, Fabian Ruehle
- Heterotic and M-theory Compactifications for String Phenomenology; L.B. Anderson