Advances in Spinor Vector duality in Calabi Yau compactifications

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- The fermionic realization of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds led to the observation of spinor-vector duality: two models are mapped to each other under the exchange of the total number of spinorial plus anti-spinorial representations and the total number of vectorial representations of an underlying $SO(2N)$ GUT symmetry group.
- This duality was proven to arise due to exchange of discrete generalised GSO phases in the free fermionic formulation
- In a bosonic representation of the spinor-vector duality the map between the dual vacua results from an exchange of a generalized discrete torsion on $\mathbb{Z}_2$ toroidal orbifolds.
- The idea is to explore the existence of similar symmetries in compactifications of heterotic string theory on smooth Calabi-Yau manifolds with vector bundles.

Main result

Any six dimensional $\mathcal{N} = 1$ supersymmetric effective field theory with the numbers of vectors N_V and of spinors N_S (of either chirality) of some $SO(2N)$ gauge group are constrained by an anomaly condition to

$$N_V = 2^{N-5} N_S + 2N - 8$$

for $N \geq 3$.

To derive this equation recall that gauge and gravitational anomalies in 6D are dictated by anomaly polynomials I_8 eight-forms For charged fermions the anomaly polynomial takes the form:

$$I_{8|R} = \widehat{A}(R_2)\,\mathrm{ch}_R(F_2)\Big|_8$$

with

$$\mathrm{ch}_R(F_2) = \mathrm{tr}_R\left[e^{i\frac{F_2}{2\pi}}\right]$$

There are three different contributions:

- N_V Hyper multiplets in the vector representation:

$$I_{8|V} \supset N_V \frac{1}{4!} \mathrm{tr}_V \left(i \frac{F_2}{2\pi} \right)^4$$

- Gauge multiplet in the adjoint representation:

$$I_{8|Ad} \supset -\frac{1}{4!} \left[(2N-8) \, \mathrm{tr}_V \left(i \frac{F_2}{2\pi} \right)^4 + 3 \left(\mathrm{tr}_V \left(i \frac{F_2}{2\pi} \right)^2 \right)^2 \right] \supset -(2N-8) \frac{1}{4!} \mathrm{tr}_V \left(i \frac{F_2}{2\pi} \right)^4$$

- N_S Hyper multiplets in the (conjugage) spinor representation:

$$I_{8|S} \supset N_S \frac{1}{4!} 2^{N-5} \left[-\mathrm{tr}_V \left(i \frac{F_2}{2\pi} \right)^4 + \frac{3}{4} \left(\mathrm{tr}_V \left(i \frac{F_2}{2\pi} \right)^2 \right)^2 \right] \supset -2^{N-5} N_S \frac{1}{4!} \mathrm{tr}_V \left(i \frac{F_2}{2\pi} \right)^4$$

Four free fermionic $T^4/\mathbb{Z}_2$ orbifold models

$$B = \{v_1, v_2, \dots, v_5\}$$

of $N_B = 5$ basis vectors defined as:

$$v_1 = 1 = \left\{\psi^{1,\dots,4}, \chi^{3,\dots,6}, y^{3,\dots,6}, \omega^{3,\dots,6} \mid \bar{y}^{3,\dots,6}, \bar{\omega}^{3,\dots,6}; \bar{\psi}^{1,\dots,6}, \bar{\eta}^{2,3}, \bar{\phi}^{1,\dots,8}\right\}$$

$$v_2 = S = \left\{\psi^{1,\dots,4}, \chi^{3,\dots,6}\right\},$$

$$v_3 = z_1 = \left\{\bar{\psi}^{1,\dots,6}, \bar{\eta}^{2,3}\right\}$$

$$v_4 = z_2 = \left\{\bar{\phi}^{1,\dots,8}\right\}$$

$$v_5 = b_1 = \left\{\psi^{1,\dots,4}, y^{3,\dots,6} \mid \bar{y}^{3,\dots,6}; \bar{\psi}^{1,\dots,6}\right\}$$

$$c\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \begin{array}{c} \beta|\alpha \\ 1 \\ S \\ z_1 \\ z_2 \\ b_1 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & S & z_1 & z_2 & b_1 \end{array} \\ \begin{pmatrix} -1 & 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 \\ -1 & -1 & 1 & \pm 1 & -1 \\ -1 & -1 & \pm 1 & 1 & \pm 1 \\ -1 & -1 & 1 & \pm 1 & -1 \end{pmatrix} \end{array}.$$

Four free fermionic $T^4/\mathbb{Z}_2$ orbifold models

$(c\left[\begin{smallmatrix} z_1 \\ z_2 \end{smallmatrix}\right], c\left[\begin{smallmatrix} b_1 \\ z_2 \end{smallmatrix}\right])$	$(+1,+1)$	$(+1,-1)$	$(-1,+1)$	$(-1,-1)$
Gauge Symmetry	$SO(4)\times SO(4)\times$ $E_7\times SU(2)\times SO(16)$	$SO(4)\times SO(4)\times$ $E_7\times SU(2)\times E_8$	$SO(4)\times SO(4)\times$ $SO(12)\times SO(4)\times SO(16)$	$SO(4)\times SO(4)\times$ $SO(12)\times SO(4)\times SO(16)$
Sector	Hyper Multiplet Representations			
S	$(4,4,1,1,1)$	$(4,4,1,1,1)$	$(4,4,1,1,1)$ $(1,1,12,4,1)$	$(4,4,1,1,1)$ $(1,1,12,4,1)$
$S\oplus(S+z_1)$	$(1,1,56,2,1)$	$(1,1,56,2,1)$		
$S+z_2$	$(1,1,1,1,128)$			
b_1				$(2_L,1,32,1,1)$ $(2_R,1,32,1,1)$
$b_1\oplus(b_1+z_1)$		$(2_L,1,56,1,1)$ $(2_R,1,56,1,1)$		
b_1+z_1	$(2_L,1,1,2_L,16)$ $(2_R,1,1,2_L,16)$	$(2_L,4,1,2,1)$ $(2_R,4,1,2,1)$	$(2_L,1,12,2,1)$ $(2_R,1,12,2,1)$ $(2_L,4,1,2,1)$ $(2_R,4,1,2,1)$	$(2_L,1,1,2_L,16)$ $(2_R,1,1,2_L,16)$
b_1+e			$(1,2_L,\overline{32},1,1)$ $(1,2_R,\overline{32},1,1)$	
$(b_1+e)\oplus$ (b_1+e+z_1)		$(1,2_L,56,1,1)$ $(1,2_R,56,1,1)$		
b_1+e+z_1	$(1,2_L,1,2_L,16)$ $(1,2_R,1,2_L,16)$	$(4,2_L,1,2,1)$ $(4,2_R,1,2,1)$	$(1,2_L,1,2_R,16)$ $(1,2_R,1,2_R,16)$	$(1,2_L,12,2_L,1)$ $(1,2_R,12,2_L,1)$ $(4,2_L,1,2_R,1)$ $(4,2_R,1,2_R,1)$
$SO(12)$ $N_V=2N_S+4$	Self–dual by E_7 enhancement	Self–dual by E_7 enhancement	$N_V=12$ $N_S=4$	$N_V=12$ $N_S=4$
$SO(16)$ $N_V=8N_S+8$	$N_V=16$ $N_S=1$		$N_V=8$ $N_S=0$	$N_V=8$ $N_S=0$

K3 Line Bundles

$$\frac{\mathcal{F}_2}{2\pi} = D_\alpha \mathrm{H}_\alpha, \quad \mathrm{H}_\alpha = V_\alpha^I \mathrm{H}_I$$

$$\mathrm{N} = -\int_{K3} \left\{ \frac{1}{2} \left(\frac{\mathcal{F}}{2\pi} \right)^2 - \frac{1}{24} \operatorname{tr} \left(\frac{\mathcal{R}}{2\pi} \right)^2 \right\} = \frac{1}{2} \kappa_{\alpha\beta} \mathrm{H}_\alpha \mathrm{H}_\beta - 2$$

Consider line bundle vectors such that the first E_8 in the ten dimensional gauge group is generically broken to $SO(10)$:

$$V_\alpha = \left(\vec{V}_\alpha \right) \left(\vec{V}'_\alpha \right), \quad \vec{V}_\alpha = \left(V_\alpha^1, V_\alpha^2, V_\alpha^3, 0^5 \right)$$

$\mathrm{N}_V = N^1_{(10)} + N^2_{(10)} + N^3_{(10)} = \frac{1}{2} \kappa_{\alpha\beta} \vec{V}_\alpha \cdot \vec{V}_\beta - 6 = c - 6$

$$N_S = N_{(16)} + N_{(16)} + N_{(16)} + N \left(\frac{3}{16)} = \frac{1}{2} \kappa_{\alpha\beta} \vec{V}_\alpha \cdot \vec{V}_\beta - 8 = c - 8$$

Indeed, inserting these expressions in this condition leads to:

$$N_V - 2^{N-5} N_S = c - 2n - 2^{N-5} 2^{n-3} (c - 8) = 8 - 2n = 2N - 8$$

using that $n = 8 - N$.

- We start with orbifold models discussed in $T^4/\mathbb{Z}_2 \times S^1$ with a Wilson line on the additional circle.
- We then consider the resolution of this orbifold to a smooth $\mathrm{K3} \times S^1$ realisation and investigate how this effects the spinor-vector duality
- In the orbifold theory $e^{2\pi i\left(V_h, P_{sh} - v_{h'} \cdot p_{sh} + v_{h'} \cdot (N - \bar{N})\right)} \cdot e^{2\pi i \frac{1}{2}\left(V_{h'} \cdot V_h - v_{h'} \cdot v_h\right)} \stackrel{!}{=} 1$ which implies

$$W \cdot P_{sh} \equiv k :$$

Or with discrete torsion
$e^{2\pi i\left(V_{h'} \cdot P_{\mathrm{sh}} - v_{h'} \cdot p_{\mathrm{sh}} + v_{h'} \cdot (N - \bar{N})\right)} \cdot e^{2\pi i \frac{1}{2}\left(V_{h'} \cdot V_h - v_{h'}, v_h\right)} \cdot e^{2\pi i \frac{1}{2}\left(kn' - k'n\right)!} = / = 1$
$W \cdot P_{sh} \equiv \frac{1}{2} k :$

Line Bundle Model with Vectorial Blowup Modes

$$V_\alpha = \left(0^2, \frac{1}{2}^6\right)\left(0^8\right)$$

for all $\alpha = 1, \ldots, 16$.

$$P_{\mathrm{sh},\alpha} = V_\alpha = V + P, \quad V = \left(\frac{1}{2}^2, 0^6\right)\left(0^8\right) \quad \text{and} \quad P = \left(-\frac{1}{2}^2, \frac{1}{2}^6\right)\left(0^8\right)$$

at all sixteen fixed points. They live on the (shifted) spinorial lattice of $SO(16)$ and part of the sixteen half-hyper multiplets $(1, 56)(1)$. Switching on these blowup modes lead to the symmetry breaking:

$$SU(2) \times E_7 \times E_8' \to SU(2) \times E_6 \times E_8'$$

In this process precisely the roots $\pm\left(0^2, 1^2, 0^4\right)\left(0^8\right)$ and $\pm\left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}^5\right)$ of the $(1, 27)(1)$ are broken.

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Line Bundle Model with Vectorial Blowup Modes

The model is compactified further on a circle S^1 with a discrete Wilson line:

$$W = \left(0^7, 1\right)\left(0^7, 1\right)$$

and the torsion phase (17) is switched off: $\epsilon = 0$. The Wilson line projection condition on the resolution is assumed to take the form:

$$W \cdot P \equiv 0$$

where P are the weights listed in Table 2 . This directly follows from the orbifold Wilson line projection (14), since the difference between the $P_{\rm sh}$ and P is at most given by V_α, but $W \cdot V_\alpha = 0$. The gauge group therefore becomes:

$$SU(2) \times SO(10) \times SO(16)'$$

and the 5D spectrum:

$$2(2,10)(1) + 36(2,1)(1) + 14(1,10)(1) + 14(1,1)(1) \ .$$

Torsion Phase (ϵ)	Without ($\epsilon = 0$)	With ($\epsilon = 1$)
Orbifold		
Gauge Group	$SU(2)_1 \times SU(2)_2 \times SO(12) \times SO(16)'$	$SU(2)_1 \times SU(2)_2 \times SO(12) \times SO(16)'$
Spectrum	$(2,2,12)(1) + 16\,\frac{1}{2}(1,2,12)(1)$	$(2,2,12)(1) + 16\,\frac{1}{2}(1,1,32)(1)$
	$+32\,(2,1,1)(1) + 4(1,1,1)(1)$	$+4(1,1,1)(1)$
Blowup		
Modes $P_{\text{sh},\alpha} = V_\alpha$	$(\frac{1}{2}, -\frac{1}{2}, 1, 0^5)(0^8)$	$(0^2, \frac{1}{2}^6)(0^8)$
Gauge Group	$SU(2) \times SO(10) \times SO(16)'$	$SU(2)_1 \times SU(2)_2 \times SU(6) \times SO(16)'$
Spectrum	$2\,(2,10)(1) + 36\,(2,1)(1)$	$2\,(2,2,6)(1) + 14\,(1,1,15)(1)$
	$+14\,(1,10)(1) + 14\,(1,1)(1)$	

Figure: This table summarises how a spinor-vector duality is visible in orbifold and resolution models. Since the resolutions depend on the choice of blowup modes, their gauge groups and therefore their spectra make this duality less apparent.

$\text{Res}(\frac{T^6}{\mathbb{Z}_2\times\mathbb{Z}_2})$

This work can be seen as a continuation of 1007.0203.

Problem

Strong triangulation dependence: 64 $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$ singular points, with four different ways to resolve each one $\longrightarrow \frac{4^{64}}{3!4!^3} \approx 4.10 \cdot 10^{33}$ different resolutions

Solution: Parameterizing Triangulations

The idea is to absorb the dependence on the triangulation in some functions s.t. we obtain expressions independent of any triangulation

- Define the following functions: $\delta^{T}_{\alpha\beta\gamma} = \begin{cases} 1 & \text{if triangulation } T \text{ is used,} \\ 0 & \text{if other triangulation is used,} \end{cases}$

  of (α, β, γ) for the four possible triangulations dubbed $T = S$, E_1, E_2 and E_3.
- Define also the following Δ functions

$$\begin{aligned} \Delta^1_{\alpha\beta\gamma} &= -\delta^{E_1}_{\alpha\beta\gamma} + \delta^{E_2}_{\alpha\beta\gamma} + \delta^{E_3}_{\alpha\beta\gamma} \ , \\ \Delta^2_{\alpha\beta\gamma} &= \delta^{E_1}_{\alpha\beta\gamma} - \delta^{E_2}_{\alpha\beta\gamma} + \delta^{E_3}_{\alpha\beta\gamma} \ , \\ \Delta^3_{\alpha\beta\gamma} &= \delta^{E_1}_{\alpha\beta\gamma} + \delta^{E_2}_{\alpha\beta\gamma} - \delta^{E_3}_{\alpha\beta\gamma} \ . \end{aligned}$$

Solution: Parameterizing Triangulations

We also obtain:

Triangl.	$\delta^{E_1}_{\alpha\beta\gamma}$	$\delta^{E_2}_{\alpha\beta\gamma}$	$\delta^{E_3}_{\alpha\beta\gamma}$	$\delta^{S}_{\alpha\beta\gamma}$	$\Delta^1_{\alpha\beta\gamma}$	$\Delta^2_{\alpha\beta\gamma}$	$\Delta^3_{\alpha\beta\gamma}$	$1-\Delta^1_{\alpha\beta\gamma}$	$1-\Delta^2_{\alpha\beta}$
E_1	1	0	0	0	-1	1	1	2	0
E_2	0	1	0	0	1	-1	1	0	2
E_3	0	0	1	0	1	1	-1	0	0
S	0	0	0	1	0	0	0	1	1

It follows immediately that

$1-\Delta^1_{\alpha\beta\gamma}-\Delta^2_{\alpha\beta\gamma}-\Delta^3_{\alpha\beta\gamma}=\delta^S_{\alpha\beta\gamma}, \quad 1-\Delta^i_{\alpha\beta\gamma}=2\delta^{E_i}_{\alpha\beta\gamma}+\delta^S_{\alpha\beta\gamma}$ and

$\Delta^2_{\alpha\beta\gamma}+\Delta^3_{\alpha\beta\gamma}=2\delta^{E_1}_{\alpha\beta\gamma}, \quad \Delta^1_{\alpha\beta\gamma}+\Delta^3_{\alpha\beta\gamma}=2\delta^{E_2}_{\alpha\beta\gamma}, \quad \Delta^1_{\alpha\beta\gamma}+\Delta^2_{\alpha\beta\gamma}=2\delta^{E_3}_{\alpha\beta\gamma}$

As a result we obtain some important results independent on the triangulation such as:

- Bianchi identities:

$$\sum_{\beta,\gamma}\mathcal{V}^2_{1,\beta\gamma}=24,\quad \sum_{\alpha,\gamma}\mathcal{V}^2_{2,\alpha\gamma}=24,\quad \sum_{\alpha,\beta}\mathcal{V}^2_{3,\alpha\beta}=24 \tag{1}$$

$$\sum_{\alpha}\left[-2+4\Delta^1_{\alpha\beta\gamma}\right]\sum_{\beta}\left[-2+4\Delta^2_{\alpha\beta\gamma}\right]\sum_{\gamma}\left[-2+4\Delta^3_{\alpha\beta\gamma}\right] \tag{2}$$

These can be further simplified to:

$$\mathcal{V}_{1,\beta\gamma}\cdot\mathcal{V}_{2,\alpha\gamma}=\mathcal{V}_{1,\beta\gamma}\cdot\mathcal{V}_{3,\alpha\beta}=\mathcal{V}_{2,\alpha\gamma}\cdot\mathcal{V}_{3,\alpha\beta}=\frac{1}{2}\left(\mathcal{V}^2_{1,\beta\gamma}+\mathcal{V}^2_{2,\alpha\gamma}+\mathcal{V}^2_{3,\alpha\beta}\right)-2 \tag{3}$$

$$\sum_{\beta}\mathcal{V}^2_{1,\beta\gamma}=\sum_{\gamma}\mathcal{V}^2_{1,\beta\gamma}=6,\quad \sum_{\alpha}\mathcal{V}^2_{2,\alpha\gamma}=\sum_{\gamma}\mathcal{V}^2_{2,\alpha\gamma}=6,\quad \sum_{\alpha}\mathcal{V}^2_{3,\alpha\beta}=\sum_{\beta}\mathcal{V}^2_{3,\alpha\beta}=6 \tag{4}$$

- Multiplicity operator:

$$\mathcal{V}_{a,\mu\nu}^2 = \frac{3}{2} \quad \Rightarrow \quad \mathcal{V}_{a,\mu\nu}\cdot\mathcal{V}_{b,\rho\sigma} = \frac{1}{4}$$

- Flux quantisation conditions:

$$2\mathcal{V}_{i,\mu\nu} \cong \sum_{\rho}\mathcal{V}_{i,\rho\nu} \cong \sum_{\rho}\mathcal{V}_{i,\mu\rho} \cong \mathcal{V}_{1,\beta\gamma} + \mathcal{V}_{2,\alpha\gamma} + \mathcal{V}_{3,\alpha\beta} \cong 0$$

- Blowup modes without oscillators condition:

$$\mathcal{V}_{a,\mu\nu}^2 = \frac{3}{2} \quad \Rightarrow \quad \mathcal{V}_{a,\mu\nu}\cdot\mathcal{V}_{b,\rho\sigma} = \frac{1}{4}$$

We also obtain other important quantities like

- Intersection numbers
- Volumes of all the curves divisors and full manifold
- Chern classes (Important consistency check):

$$c_3 = \frac{1}{4}\sum_{i,\alpha,\beta,\gamma}\left(1+\Delta^i_{\alpha\beta\gamma}\right) - \frac{1}{4}\sum_{i,\alpha,\beta,\gamma}\left(-1+\Delta^i_{\alpha\beta\gamma}\right) = 96$$

Note in particular that all the triangulation dependence in the form of the functions $\Delta^i_{\alpha\beta\gamma}$ drop out and the final result equals the known Euler number 96 .

Res$(\frac{T^6}{\mathbb{Z}_2\times\mathbb{Z}_2})$: Models

Without Wilson lines

- Models with three bundle vectors
- $SO(10) \times SO(12)$ Line Bundle Models
- Blaszczyk's $SU(3) \times SU(2)$ Line Bundle Models
- A "swampland" $SO(10) \times SO(10)$ models

With one Wilson line

- Models with to Two Sets of Three Independent Line Bundles
- $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ generalisation of the $T^4/\mathbb{Z}_2 \times S^1$ model