

Ecole Polytechnique

Based on works in collaboration with :

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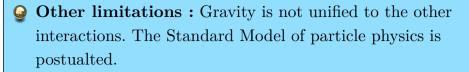
Liverpool, May 30, 2012

## Standard cosmology

- $\bigcirc$  The cosmological evolution is described by the  $\ \Lambda\text{-CDM}$  model :
  - Inflation : Dilutes inhomogeneities, solves horizon, flatness problems...
  - Q Reheating followed by phase transitions (electroweak, QCD confinement) ➡ radiation and matter dominated eras.
  - $\bigcirc$   $\Lambda$ -dominated era :  $\Lambda \geqq 0$

#### **Q** Limitations :

- *Q* In General Relativity : Gravity is not quantized. *Q*
- General Big bang singularity.
- Phenomenological approach : GR is coupled to fluids, with state equations  $P_i = w_i \rho_i$  and % of  $\rho_i$  imposed to fit observations
  - $\sqrt{w_i} = 1/3$  for radiation,
  - $v w_i = 0$  for matter (Dark = 27\%, SM = 3\%)
  - $\sqrt{w_i} = -1$  for Dark Energy  $\Lambda$  (70%).
- Phase transitions between eras imposed by hand.



Gear Anticomposition → Ant

✓ Large number of parameters to fix.

Hierarchie problem : at 1-loop, the Higgs mass is attracted to a very large UV cut-off.

Q Extensions such as the Minimal Supersymmetric Standard Model : supersymetry is « softly broken ».

[Fayet]

 $\checkmark$  Need to explain the hierarchy  $M_{\rm susy} \ll M_{\rm Planck}$ .

# Stringy approach

- - $\bigcirc$   $\Leftrightarrow$  Describes quantum gravity consistently.
  - $\bigcirc$   $\Leftrightarrow$  hope : Describe big bang without initial singularity.
  - Our Derivation of the nature of the cosmic fluids and their state equations.
  - Output interactions of the second second
  - General General Hope : Describe the phases of the universe and the transitions between them in a single and coherent theoretical context.

#### **Q** How can we approach such a program?

- It is natural to look for string models, which describe at tree level time-dependent backgrounds. But not so many are known!!
- Generically, one finds Anti-de Sitter or Minkowski universes : Admit time-independent metrics : Static
- $\bigcirc$  Since we want at the end a small and positive cosmological constant, let us focus on the case  $\Lambda_{tree} = 0$ .

#### $\bigcirc$ At 1-loop :

- $\bigcirc$  The non-supersymmetric models will generate  $\Lambda_{1-\text{loop}}$ .
- The latter backreacts on the originally static background : A quasi static evolution emerges.
- $\bigcirc$  The induced cosmology is a pure quantum effect !

### **Q** Non susy models :

- $\bigcirc$  If there is no susy at all :  $\Lambda_{1\text{-loop}}$  of order  $\pm 1$
- $\bigcirc$  For  $\Lambda_{1-\text{loop}}$  to be as small, we focus on models where susy is spontaneously broken at tree level (super-Higgs mechanism).
- $\bigcirc$  The scale  $M_{susy}$  of susy breaking is a scalar field, which may evolve in time.
- $\bigcirc$  It can decrease and generate the hierarchy  $M_{\rm susy} \ll M_{\rm Planck} \, .$
- $\bigcirc$  Simplest realization is a susy model at finite temperature T.
  - $\bigcirc$  For realistic models : break spontaneously susy at scale  $M_{\rm susy}$  and switch on T .

Some models describe a bounce : contraction $\rightarrow$ expansion with no singularity at the transition. $T, M_{susy}$ and the string coupling $g_{st}$ decrease as the universe expands.	HAGEDORN ERA $T \approx M_{\text{string}}$	INTERMEDIATE ERA $M_{ m ew} < T < M_{ m string}$	$\frac{\text{STANDARD}}{\text{COSMOLOGY}}}{T < M_{\text{ew}}}$
$T,~M_{ m susy}$ and the string coupling $g_{ m st}$ decrease as the universe			$n \rightarrow expansion with$
	no singularity at the t	Tansmon.	
	expands.		
Electroweak radiative breaking when $T \approx M_{\text{ew}} \Rightarrow \langle M_{\text{susy}} \rangle \approx M_{\text{ew}}$ .	expands. $M_{ m susy}$ and $g_{ m st}$ are exp	ected be stabilized by	IR effects :

## Moduli stabilization

- - Would mediate long range force, in contradiction with precision tests on the gravitational force.
  - Gauge couplings and masses depend on the moduli VEV's : The theory loses its predictability.
- Q Litterature : Solve this problem in susy models. The moduli remain massless to all order in perturbation theory ➡ Non-perturbative effects to break susy and give masses : Difficult!!
  - Solution When susy broken at tree level, except for  $M_{susy}$  and  $g_{st}$ , the moduli admit a potential at 1-loop. Acquire masses and are stabilized at local minima.

#### **Q** This is non trivial : **Cosmological moduli problem**

♀ If they acquire a mass, they will oscillate around their minima

 $\dot{\rho}_{
m mod} + 3H(\rho_{
m mod} + P_{
m mod}) = 0 \implies \rho_{
m mod} \propto \frac{1}{a^3} \gg \frac{1}{a^4}$  (for radiation)  $P_{
m mod} \simeq 0$  (for dust)

which overcloses the universe (except if oscillations are unaturally tuned to be very small).

- If they are massive enough, they can decay into gauge bosons before nucleosynthesis (in order to not alter the abundances of <sup>4</sup>Helium and Deterium) :
- $\bigcirc$  ➡  $M_{\text{mod}} > 10 \text{ TeV}$ , which is in contradiction with  $M_{\text{mod}} = O(M_{\text{susy}}) \approx 1 \text{ TeV}$  to solve the hierachy problem.

 $M_{mod}(t) \propto T(t) \implies 
ho_{
m mod} \propto 1/a^4$ 

i.e. no need to decay into radiation !!

♀ Only at the exit of the intermediate era, when  $T(t) \approx M_{\text{ew}}$ , the masses acquire their final constant values  $M_{\text{mod}} = O(M_{\text{susy}}) \approx 1$  TeV.



- **Q** String thermodynamics.
- **Q** State equations and induced cosmology.
- **Q** Moduli stabilization in perturbative heterotic string.
  - $\bigcirc$  At finite temperature T only (no  $M_{susy}$ ).
  - $\bigcirc$  With both  $M_{susy}$  and T.
- $\bigcirc$  Including D-branes (solitonic states) for the open string case (finite T only).
- $\bigcirc$  Calabi-Yau compactifications in type II string (finite T only).

### String Thermodynamics

 $\bigcirc$  In quantum statistical physics,  $\rho$  and P can be evaluated from the canonical partition function  $\mathcal{Z}_{th} = \operatorname{Tr} e^{-\beta H}$ .

 $\bigcirc$  At weak coupling *i.e.* for a perfect gas, the free energy for a

free field is

$$F = -\frac{\ln \mathcal{Z}_{\rm th}}{\beta} = \frac{1}{2\beta} \operatorname{tr} \ln(-\Box + \mu^2)$$

It can be evaluated and rewritten in 1<sup>st</sup> quantized formalism : The virtual loop of the particle wraps  $\tilde{m}_0$  times  $S^1(R_0)$ .

$$F = -(-)^{F} \int_{0}^{+\infty} \frac{dl}{2l} \frac{V_{\text{box}}}{(2\pi l)^{\frac{D-1}{2}}} e^{-\frac{\mu^{2} l}{2}} \frac{1}{\sqrt{2\pi l}} \sum_{\tilde{m}_{0}} e^{-\frac{\beta^{2} \tilde{m}_{0}^{2}}{2l}} (-)^{F\tilde{m}_{0}}$$

$$= -\frac{Z}{\beta} \text{ where } Z \text{ is the vacuum-to-vacuum amplitude}$$
(the 1-loop partition function in the Euclidean background)
$$\bigcirc \text{ The same amplitude in string theory reads}$$

$$-\frac{Z}{\beta} = -(-)^{F} \int_{\mathcal{F}} \frac{d\tau_{1} d\tau_{2}}{2\tau_{2}} \frac{V_{\text{box}}}{(2\pi\sqrt{\tau_{2}})^{D-1}} \sum_{\text{spectrum}} e^{-\pi\mu^{2}\tau_{2}}$$

$$-\frac{1}{2\pi\sqrt{\tau_{2}}} \sum_{n_{0},\tilde{m}_{0}} e^{-\frac{\pi R_{0}^{2}}{\tau_{2}} |n_{0}\tau + \tilde{m}_{0}|^{2}} (-)^{F\tilde{m}_{0} + \tilde{F}n_{0} + \tilde{m}_{0}n_{0}}$$

$$\bigcirc \text{ Modular invariance :}$$

$$\int_{\mathcal{F}} d\tau_{1} d\tau_{2} \sum_{n_{0},\tilde{m}_{0}} \longrightarrow \int_{0}^{+\infty} d\tau_{2} \int_{-1/2}^{1/2} d\tau_{1} \sum_{\tilde{m}_{0}} \text{ with } n_{0} = 0$$

$$\Leftrightarrow \text{ Free energy for the infinite set of string oscillation modes.}$$

## Induced cosmology

 ${\it Q}$  At weak coupling, the string effective supergravity action at finite T contains a tree level part (kinetic terms) + the 1-loop vacuum-to-vaccum amplitude Z computed in the Euclidean background  $S^{1}(R_{0}) \times T^{D-1}(R_{\text{box}})$ 

$$S = \int d^{D}x \sqrt{-g} \left[ e^{-2\phi} \left( \frac{R}{2} + 2(\partial\phi)^{2} + \cdots \right) + \frac{Z}{\beta V} \right]$$
$$\frac{1}{g_{\rm st}^{2}}$$

 $\bigcirc$  At the tree level, the metric is the analytic continuation of that of  $S^1(R_0) \times T^{D-1}(R_{\text{box}})$ 

$$ds^{2} = -\beta^{2} dt^{2} + a^{2} \left( dx^{1^{2}} + \dots + dx^{D-1^{2}} \right)$$
  
where  $\beta = 2\pi R_{0}$ ,  $a = 2\pi R_{\text{box}}$ 

ere 
$$\beta = 2\pi R_0$$
,  $a = 2\pi R_{\text{box}}$ 

- **Q** Looking for homogeneous and isotropic evolutions :
  - $\bigcirc$  All quantities depend only on time.
  - $\begin{aligned} & \varTheta \text{ Varaying the metric } \Rightarrow \quad T^{(1) \ \nu}_{\ \mu} = \text{diag}(-\rho, P, \dots, P)_{\mu}^{\ \nu} \\ & \rho = -\frac{1}{V} \frac{\partial Z}{\partial \beta} \equiv \frac{1}{V} \left(\frac{\partial(\beta F)}{\partial \beta}\right)_{V,\phi,\dots} \\ & P = \frac{1}{\beta} \frac{\partial Z}{\partial V} \equiv -\left(\frac{\partial F}{\partial V}\right)_{\beta,\phi,\dots} \end{aligned}$

These relations are not derived by postulating the 1<sup>st</sup> et 2<sup>d</sup> low of thermodynamics.

The state equation  $P/\rho = w$  is not postulated but derived from the microscopic evaluation of Z.

 $\checkmark w$  depends on all moduli fields and thus on time.

## Heterotic string at finite T

- $\bigcirc Z$  is computed in  $S^1(R_0) \times T^{D-1}(R_{ ext{box}}) \times T^{10- ext{D}}$
- $\bigcirc$  The 1-loop effective action in Einstein frame is

$$S = \int d^D x \sqrt{-g} \left[ \frac{R}{2} - \frac{2}{D-2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} F_{MN}(\Phi^P) \partial_\mu \Phi^M \partial^\mu \Phi^N - \mathcal{F} \right]$$
  
$$\Phi^M : \text{ metric } \hat{g}_{ij}, \text{ antisymmetric tensor } \hat{B}_{ij}, \text{ Wilson lines } Y_i$$

$$\mathcal{F} = -T^D \sum_s G\left(rac{e^{rac{2}{D-2}\phi} \hat{M}_s(\Phi^M)}{T}
ight)$$

where  $e^{\frac{2}{D-2}\phi}\hat{M}_s$  and T are the masses and temperature measured in Einstein frame.

 $\Rightarrow$  The free energy density  $\mathcal{F}$  is an effective potential at finite temperature for the moduli !

$${\cal F}=-T^D\sum_s Gigg({e^{{2\over D-2}\phi}\hat{M}_s(\Phi^M)\over T}igg)$$

 $\bigcirc$  G is picked for zero masses :

$$G(0) = \sigma$$
 Stefan-Botlzmann constant

$$G(x) \sim \left(\frac{x}{2\pi}\right)^{\frac{D-1}{2}} e^{-x}$$
 when  $x \gg 1$ 

 $\Rightarrow \mathcal{F} \text{ admits a local minima at any } \Phi_0^M \text{ such that some masses vanish. This corresponds to enhanced gauge symmetry points.}$  $\bigcirc \text{ In the neighborhood of } \Phi_0^M,$ 

$$\mathcal{F} = -T^D \left\{ n \,\sigma + \sum_{u=1}^{n_0} G\left(\frac{e^{\frac{2}{D-2}\phi} \hat{M}_u(\Phi^M)}{T}\right) + O\left(e^{-\frac{\hat{M}_{\min}}{\hat{T}}}\right) \right\}$$

where the expontentially suppressed terms can be neglected for low enough temperature. Q In this regime, the eqs of motion admit a particular solution a(t) ∝ 1/(T(t)) ∝ t<sup>2/D</sup>, φ(t) ≡ φ<sub>0</sub> arbitrary, Φ<sup>M</sup>(t) ≡ Φ<sub>0</sub><sup>M</sup>
which is a radiation era, H<sup>2</sup> ∝ 1/(a<sup>D</sup>) with Φ<sup>M</sup> sitting at a minimum.
Q Is this solution « unique » in the sense it is an attractor ?
Φ<sup>M</sup> = Φ<sub>0</sub><sup>M</sup> + ϵ<sup>M</sup> ⇒ ϵ<sup>M</sup> + (D − 1)H ϵ<sup>M</sup> + Λ<sup>M</sup><sub>N</sub> ϵ<sup>N</sup> = 0
where Λ<sup>M</sup><sub>N</sub> ∝ T<sup>D−2</sup> *i.e.* time -dependent masses :
M<sub>mod</sub>(t) ∝ T(t) <sup>D-2</sup>/<sub>2</sub> ⇒ 1/2 F<sub>MN</sub> ϵ<sup>M</sup> ϵ<sup>N</sup> ∝ 1/(a<sup>3D/2−2</sup>)
Q For D ≥ 5, it is ≪ 1/(a<sup>D</sup>) *i.e.* radiation dominated: attractor
Q For D = 4, it is ∝ 1/(a<sup>D</sup>) *i.e.* radiation-like With  $M_{susy}$  and T

Q Heterotic string on  $\frac{T^6}{\mathbb{Z}_2 \times \mathbb{Z}_2}$  is  $\mathcal{N} = 1$  in D = 4.

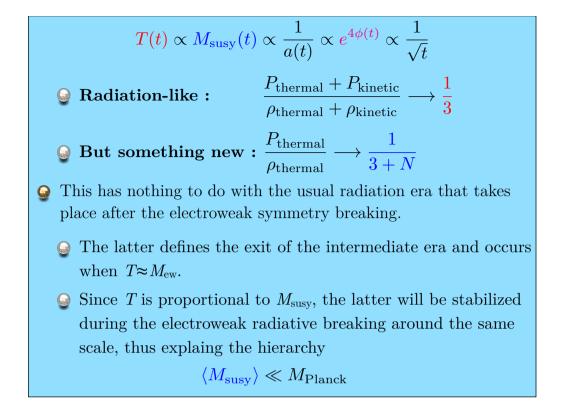
**Q** Introduce non-trivial boundary conditions along circles to break susy and switch on finite temperature

$$T = \frac{1}{2\pi R_0} \qquad \qquad M_{\text{susy}} = \frac{1}{2\pi (\prod_i^N R_i)^{1/N}}$$

- $\bigcirc$  Both T(t) and  $M_{susy}(t)$  are running away, but their ratio  $M_{susy}/T$  admits a minimum of  $\mathcal{F}$  of order 1.
- ♀ Attraction to the « unique » solution

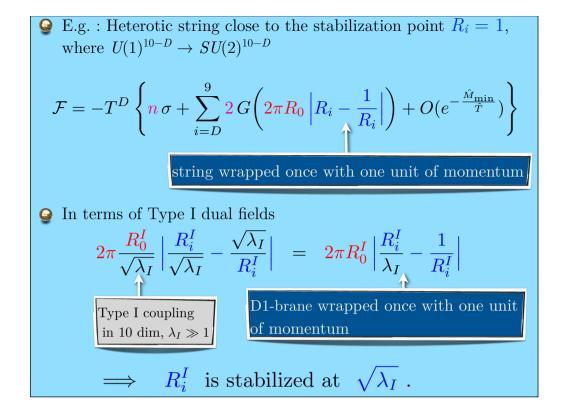
$$T(t) \propto M_{
m susy}(t) \propto rac{1}{a(t)} \propto e^{4\phi(t)} \propto rac{1}{\sqrt{t}}$$

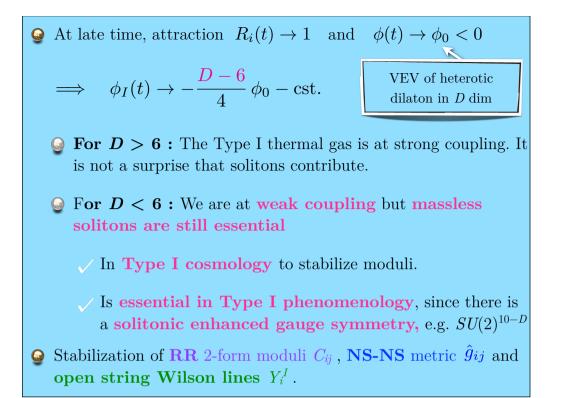
with other moduli stabilized as before.

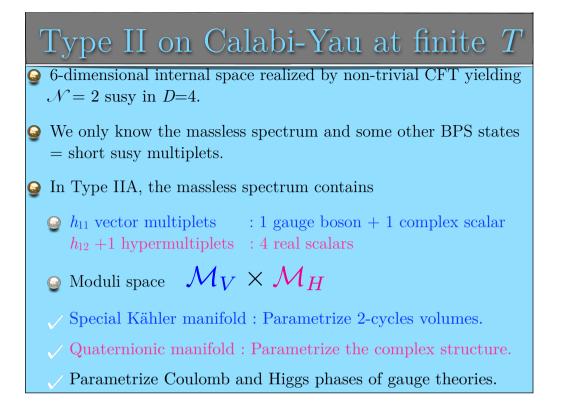


# Type I string at finite T

- **Q** Contains closed strings and open strings.
- $\Theta$  As in heterotic, we consider  $S^{1}(R_{0}) \times T^{D-1}(R_{\text{box}}) \times T^{10-D}$ .
- $\bigcirc$  No enhanced symmetry point  $\rightleftharpoons$  No moduli stabilization ?
- $\bigcirc$  The Heterotic and Type I strings are dual (S-dual in D=10).
  - $\checkmark$  Their respective gases at finite T must be dual.
    - Since the backreactions we study are **quasi-static**, we can use **Heterotic** / **Type I duality to derive the Type I cosmology at finite** *T* at each instant *t*.







### $\mathcal{M}_V imes \mathcal{M}_H$

- $\bigcirc$  Dilaton is in a hypermultiplet  $\vartriangleleft$  metric of  $\mathcal{M}_H$  is corrected.
- $\mathbf{Q}$  That of  $\mathcal{M}_V$  is exact at tree level.
  - *Q* It is singular where the volume of 2-cycles vanish. *Q* − 2-cycles vanish.
  - D2-branes (= membranes) wrapped on them realize hypermultiplets becoming massless. [Strominger]
  - $\bigcirc$  Integrated out from effective action, they induce an IR logarythmic divergence of the  $\sigma$ -model action.
- $\Theta$  We want to show that at finite *T*, the moduli in  $\mathcal{M}_V \times \mathcal{M}_H$  are stabilized at points where these solitons become massless.

(I) In the vincinity of a singular point  $P_0$  in  $\mathcal{M}_V$ , we identify the gauge group and charged matter arising from D2-branes wrapped on the vanishing 2-cycles.

(II) These solitonic objects are electrically charged under the gauge group. We can write an effective action that includes them as elementary fields, in addition to the massless perturbative string states (they are neutral).

(III) At weak coupling, the tree level part is a  $\sigma$ -model based on  $\tilde{\mathcal{M}}_V \times \tilde{\mathcal{M}}_H$ , whose metric is non-singular and admits isometries we can gauge in order to reproduce the gauge theory.

(IV) The gauging  $\Rightarrow$  tree level potential  $\mathcal{V}$  we determine around  $P_0$ . - Coulomb branch = compactification on the original CY. - Higgs branch = compactification on a distinct CY'. Extremal transition CY  $\rightarrow$  CY' corresponds to replacing the vanishing 2-spheres by 3-spheres. (V) In each branch,  $\mathcal{V} \Leftrightarrow$  tree level masses that can vanish. They depend on the moduli which parametrize  $\tilde{\mathcal{M}}_V \times \tilde{\mathcal{M}}_H$ .

(VI) At 1-loop action = Tree level action evaluated in a branch + the associated 1-loop correction at finite T,  $\mathcal{F}(\text{Masses}(moduli))$ 

**(VII)** Backreaction  $\Rightarrow$  radiation-like era with moduli attracted to the origin (=intersection) of Coulomb and Higgs branches. Their 1-loop masses are of order T(t). The internal CY stays at the extremal transition.

### **Q** Stabilization at a conifold locus :

- $\bigcirc R$  2-spheres vanish in CY  $\Rightarrow R$  hypermultiplets
- $\bigcirc$  But in S homology classes  $\Leftrightarrow$  charged under  $U(1)^S$
- $\bigcirc$  When the D2's become massless, they can acquiere VEV's

and Higgs  $U(1)^S$ . The hypers « not eaten » become perturbative moduli of CY'

$$h_{11}' = h_{11} - S$$
  $h_{12}' = h_{12} + R - S$ 

 $\bigcirc$  Tree level action in the neighborhood of the conifold locus

$$S_{\text{tree}} = \int d^4x \sqrt{-g} \left\{ \frac{R}{2} - g_{I\bar{J}}^{(0)} \partial_\mu X^I \partial^\mu \bar{X}^J - \frac{1}{2} \nabla_\mu c^{\mathcal{A}u} \nabla^\mu c^{\mathcal{A}u} - h_{\alpha\beta}^{(0)} \partial_\mu q^\alpha \partial^\mu q^\beta - \mathcal{V} + \cdots \right.$$
$$\mathcal{V} = e^{\mathcal{K}^{(0)}} \left( 2 Q_i^{\mathcal{A}} Q_j^{\mathcal{A}} \bar{X}^i X^j \mathcal{C}^{\mathcal{A}\dagger} \mathcal{C}^{\mathcal{A}} + \frac{1}{4} g^{(0)i\bar{j}} D_i^x D_j^x \right)$$
$$\mathcal{C}^{\mathcal{A}} = \begin{pmatrix} i(c^{\mathcal{A}1} + ic^{\mathcal{A}2}) \\ (c^{\mathcal{A}3} + ic^{\mathcal{A}4})^* \end{pmatrix} \qquad D_i^x \equiv Q_i^{\mathcal{A}} \mathcal{C}^{\mathcal{A}\dagger} \sigma^x \mathcal{C}^{\mathcal{A}}$$

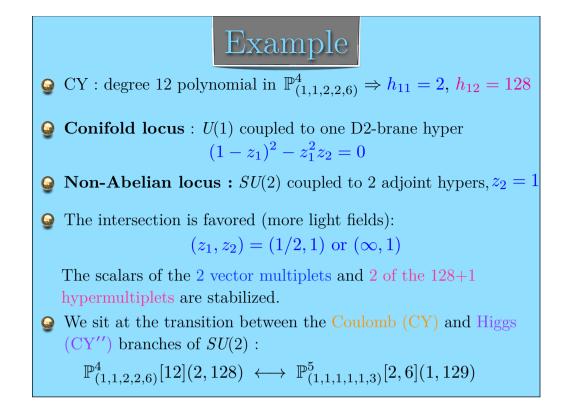
Q Coulomb branch: X<sup>i</sup> arbitrary, c<sup>Au</sup> = 0 (original CY)
✓ The D2-branes masses are M<sup>2</sup><sub>A</sub> = 4 e<sup>K<sup>(0)</sup></sup> |Q<sup>A</sup><sub>i</sub>X<sup>i</sup>|<sup>2</sup> + ···
✓ The 1-loop free energy F is minimal when they vanish
➡ X<sup>i</sup> = 0 i.e. CY → conifold configuration
Q Higgs branch: X<sup>i</sup> = 0, c<sup>Au</sup> such that D<sup>x</sup><sub>i</sub> = 0 (CY')
✓ F is minimal when the masses of the S Higgsed vector multiplets vanish ➡ c<sup>Au</sup> = 0 i.e. conifold configuration ← CY'
Q At the conifold locus, the vector multiplets scalars X<sup>i</sup> and charged hypermultiplets c<sup>Au</sup> aquiere a 1-loop mass proportional to T(t), while U(1)<sup>S</sup> remains massless.

### **Q** Stabilization at a non-Abelian locus :

- $\bigcirc$  CY can develop a genus-g curve of  $A_{N-1}$  singularities, realized by N-1 vanishing 2-spheres.
- $\bigcirc$  Wrapped D2-branes enhance  $U(1)^{N-1}$  to SU(N), with g hypermultiplets in the ajoint.
- Gereg ≥ 2, this theory is non-asymptotically free and admits 2 branches : Coulomb ( $U(1)^{N-1}$  on CY), Higgs (on CY'')

$$h'_{11} = h_{11} - (N-1)$$
  $h'_{12} = h_{12} + (g-1)(N^2 - 1) - g(N-1)$ 

Q Coulomb branch:
Cartan (X<sup>i</sup>, c<sup>iAu</sup>) arbitrary, non-Cartan X<sup>â</sup> = c<sup>âAu</sup> = 0
✓ The Cartan's are stabilized where the tree level masses of the non-Cartan vector and hypermultiplets vanish
➡ X<sup>i</sup> = c<sup>iAu</sup> = 0 i.e. singular CY
Q Higgs branch: X<sup>a</sup> = 0, c<sup>aAu</sup> such that D<sup>bx</sup> = 0
✓ The c<sup>aAu</sup> are stabilized where the tree level masses of the SU(N) vector multiplets vanish
➡ c<sup>aAu</sup> = 0 i.e. singular CY''
Q At the non-Abelian locus, the vector multiplets scalars X<sup>a</sup> and charged hypermultiplets c<sup>aAu</sup> obtain a 1-loop mass proportional to T(t), while SU(N) remains massless.



- $\bigcirc$  The heterotic dual on K3× $T^2$  is known : The torus modulus  $T_h$  and the axio-dialton  $S_h$  are stabilized, at strong coupling.
- $\bigcirc$  In general: Any Type II compactication on a CY at finite T
  - $\bigcirc$  We expect  $\mathcal{M}_V$  to be lifted : In IIA, all 2-cycles vanish.
  - $\bigcirc$  The moduli in  $\mathcal{M}_H$  associated to 3-cycles that can vanish and be resolved into 2-cycles should be lifted.
  - Q This is not the case for all, in particular the universal hypermultiplet which contains the Type II dilaton.

### Summary

- **Q** The cosmological evolution can be seen as the backreaction of quantum and thermal corrections on flat Minkoski space.
- $\bigcirc$  In a string context, there are at least 3 eras : We have focussed on the Intermediate one :  $M_{\text{string}} < T < M_{\text{ew}}$ 
  - String theory provides a microscopic derivation of the cosmic fluids, their state equations.
  - $\bigcirc$  Attraction to a radiation-like evolution which brings the universe towards the electroweak phase transition, while generating the hierarchy  $M_{susy} \ll M_{Planck}$ .
  - *Q* Provides a mechanism for moduli stabilization.