Time in Quantum Mechanics

Observables correspond to operators (or POVMs) buuut time in QM is just an incremented parameter like $\hbar$ and m.

Any child with a stop-watch can tell you time is an observable quantity but it turns out to be an awkward thing to define as quantum objects are 'fuzzy', capable of travelling backwards and having poorly defined locations.

Quantum states (/operators in Heisenberg picture) are time-dependent and evolve under Schrödinger equation, but this implies arbitrary precision for time. This is contrary to how QM observables function.

But how do we measure classical time? By linking to another physical quantity (pulses, position of clock hand). Uncertainty in these $\Rightarrow$ uncertainty in time. Maybe a 'time' operator needs to be system dependent?

First up, history of time in QM

'How can we say ~~a~~ when a particle crosses a point?'

A Pauli's argument

If a self-adjoint time operator existed $T = T^\dagger$

could define

$$U = e^{iE'T/\hbar} \quad \text{s.t.} \quad U|E\rangle = U|E - E'\rangle$$

Which of course permits negative energies and continuous spectra

"the introduction of an operator T must fundamentally be abandoned..."

(footnote of his encyclopaedia)

One statement from such a renown man killed it off forever.

note - can do QM without self-adjoint (Hermitian) operators, but all ~~popular ones~~ mainstream ones do.

First attempt would be a QM version of $t = \frac{mx}{p}$

$$T = \frac{m}{2}(xp^{-1} + p^{-1}m)$$

$\boxed{\text{gives the time a particle travels } \sout{a} \text{ to a particular point (say origin); time of arrival}}$

however the interpretation of said operator is lacking, especially in regards to an uncertainty relation (remember this is old fashioned QM)

$\Delta E \Delta t$ like relation holds if

ΔE - uncertainty in E
Δt - lifetime of state

but <u>not</u>

ΔE - energy transferred
Δt - duration of measurement

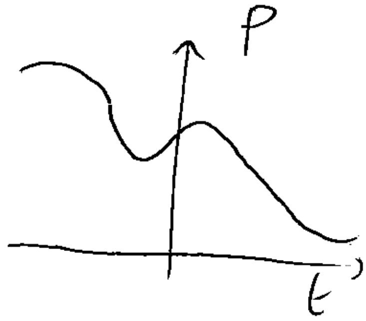
And for T earlier it has no meaning either. Though a modern approach may ignore this though (as ΔE could be 0 for our travelling particle arbitrary.)

+ Backflow! Quantum particles can go backwards

$$J = \frac{\hbar}{2mi}\left(\psi^* \frac{d\psi}{dx} - \psi \frac{\partial \psi^*}{\partial x}\right) = J(x,t)$$

s.t. $F(t_1, t_2) = \int_{t_1}^{t_2} dt\, dx\, J(x=0,t)$

flux of probability going through origin.



(Can also write in terms of Wigner function.)

J does not have to be positive in a region (though overall it must be $F(\infty,\infty) > 0$ as right moving)

Interpret $v = \frac{J}{|\psi|^2}$, so particle has negative velocity

Next up a density-based one

$$\Pi^K(t) = \left| \int_0^\infty dp \left(\frac{p}{2\pi m\hbar}\right)^{1/2} e^{-\frac{ip^2t}{2m\hbar}} \psi(p) \right|^2$$
$$+ \int_{-\infty}^{0} dp \left(\frac{-p}{2\pi m\hbar}\right)^{1/2} e^{-\frac{ip^2t}{2m\hbar}} \psi(p) \Big|^2$$

probability density that reproduces same statistics as above (average value) but different densities / particular value for a given t.

So what goes wrong?

+ Quantum-classical interface (measuring device) and our interpretations of causality.
  A common issue in QM of course

+ Badly defined quantities and concepts.
  - dwell time (between two points)
  - arrival time
  - ordering.

+ ~~Backflow~~ Focus on olde time QM
  - uncertainty like $\Delta E \Delta t \geq \frac{\hbar}{2} \iff [t, H] = i\hbar$

  but what does $\Delta E, \Delta t$ mean?

$$W(p,x) = \frac{1}{\hbar\pi}\int_{-\infty}^{\infty} dy\, e^{\frac{ipy}{\hbar}} \psi^*(x+\tfrac{y}{2})\psi(x-\tfrac{y}{2})$$

s.t. $\langle A \rangle = \int dp\,dx\, W A$

$$J(x) = \int dp\,dq\, \frac{p\delta(q)}{m} W(p,q)$$

so ~~some sign~~ depending on p can get negative J.

encodes all exp. values into phase space

Amount of backflow is bounded spatially & temporally.

States like

$\psi(p) = (a-p)f(p)$ give surprisingly good backflow

such as Gaussian-like (and thus experimentally realisable)

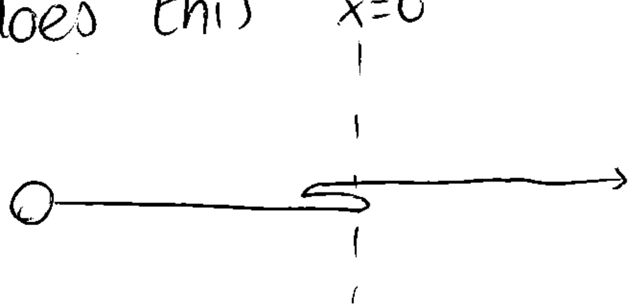
$\psi = (a-p)\Theta(p)e^{-bp^2}$

which adding a tail can look quite Gaussian.

Relative values of a & b control it.

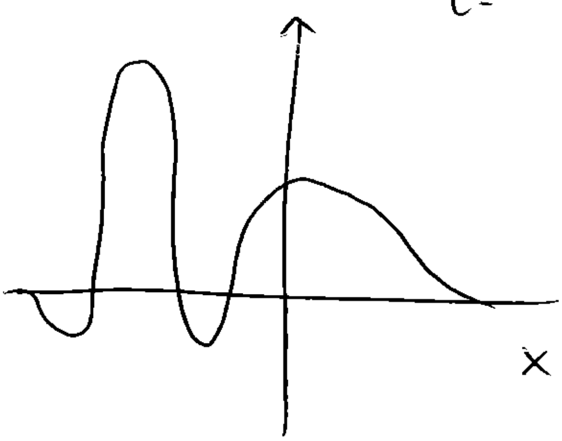
Note could scale it all to be arbitrarily wide as well, ~~it's only~~ a free choice for x.

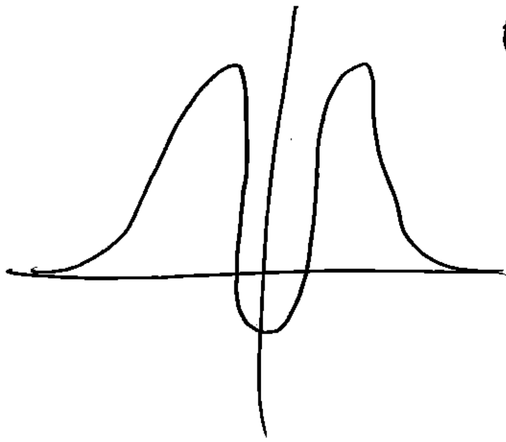
This arbitrary space dies considering smearing functions representing actual measurements particle does this

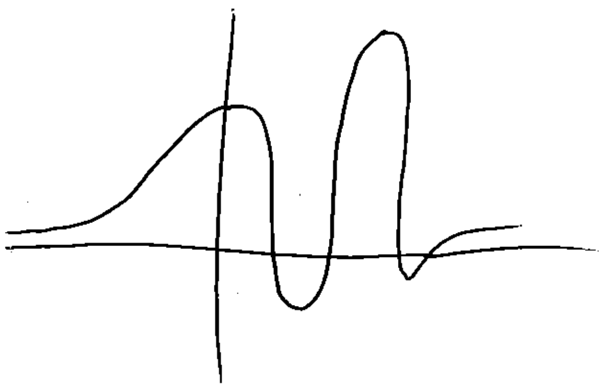


max is 4% prob. going backwards

J

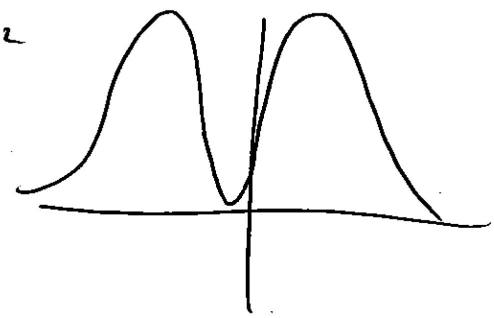


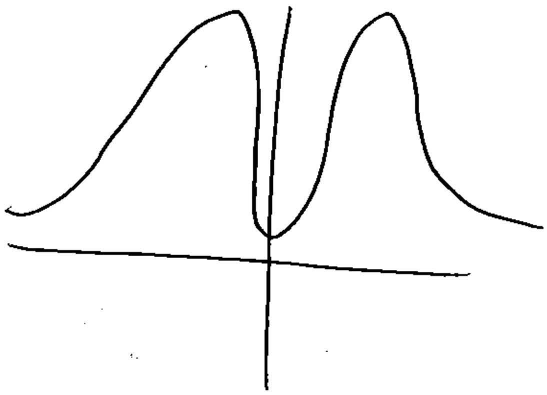




numerically integrated ~~[illegible]~~ and it is a small change

$|\psi|^2$





<u>not</u> symmetric.

area on LHS slips back a bit

so maybe the conclusion is that the time of arrival just doesn't work (same for dwell time) as it can go backward (fuzziness of QM)

+ Using Bohmian pilot wave interpretation particle actually does move backward.

+ total number of backflow is independent of $\hbar$ so it would happen classically? But ~~disap~~ $\hbar$ dependence returns after considering proper measurement

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"Quantum Zeno effect"

repeated measurements lock a system into a particular state

so you cannot constantly measure the position of a particle without affecting it (removes that arrival time loop hole)

Can replicate ~~th~~ repeated measurements by complex potential done every time interval ϵ

P is our projection $\qquad \bar{P} = 1 - P$

$Pe^{-iH\epsilon}Pe^{-iH\epsilon}$ $\qquad P|\psi\rangle \approx e^{-i(H - iV\bar{P})t}|\psi\rangle$

$e^{-V\bar{P}\epsilon} = P + e^{-V\epsilon}\bar{P}$

$P \approx e^{-V\epsilon\bar{P}} \Rightarrow PePe \approx e^{(-iH - V_P)n\epsilon}$

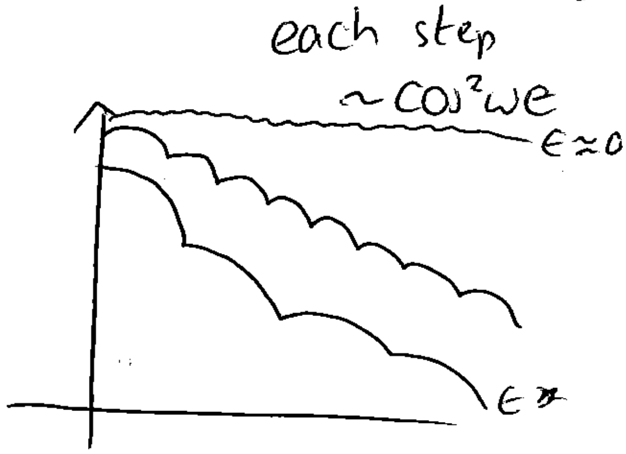
$n\epsilon = t$

corresponds to adding potential $= iV(1-P)$

$= iV\bar{P}$

$\epsilon \propto 1/V$

can be ~~like~~ imagined as complex anti-measurements stealing probability into complex plane.

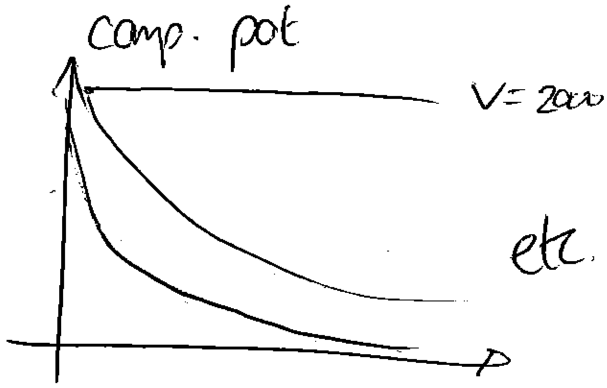


Of course

$P = |\uparrow\rangle\langle\uparrow|$, $H = \omega\sigma_x$

$\Rightarrow \hat{V} = iV(1 - |\uparrow\rangle\langle\uparrow|)$

$= iV(|\downarrow\rangle\langle\downarrow|)$



etc.

~~jnsta~~ Not massive profound but quite interesting considering complex valued potentials 'are not allowed'