Questions of Causality in Quantum Field Theory

University of Liverpool, 25th November 2014

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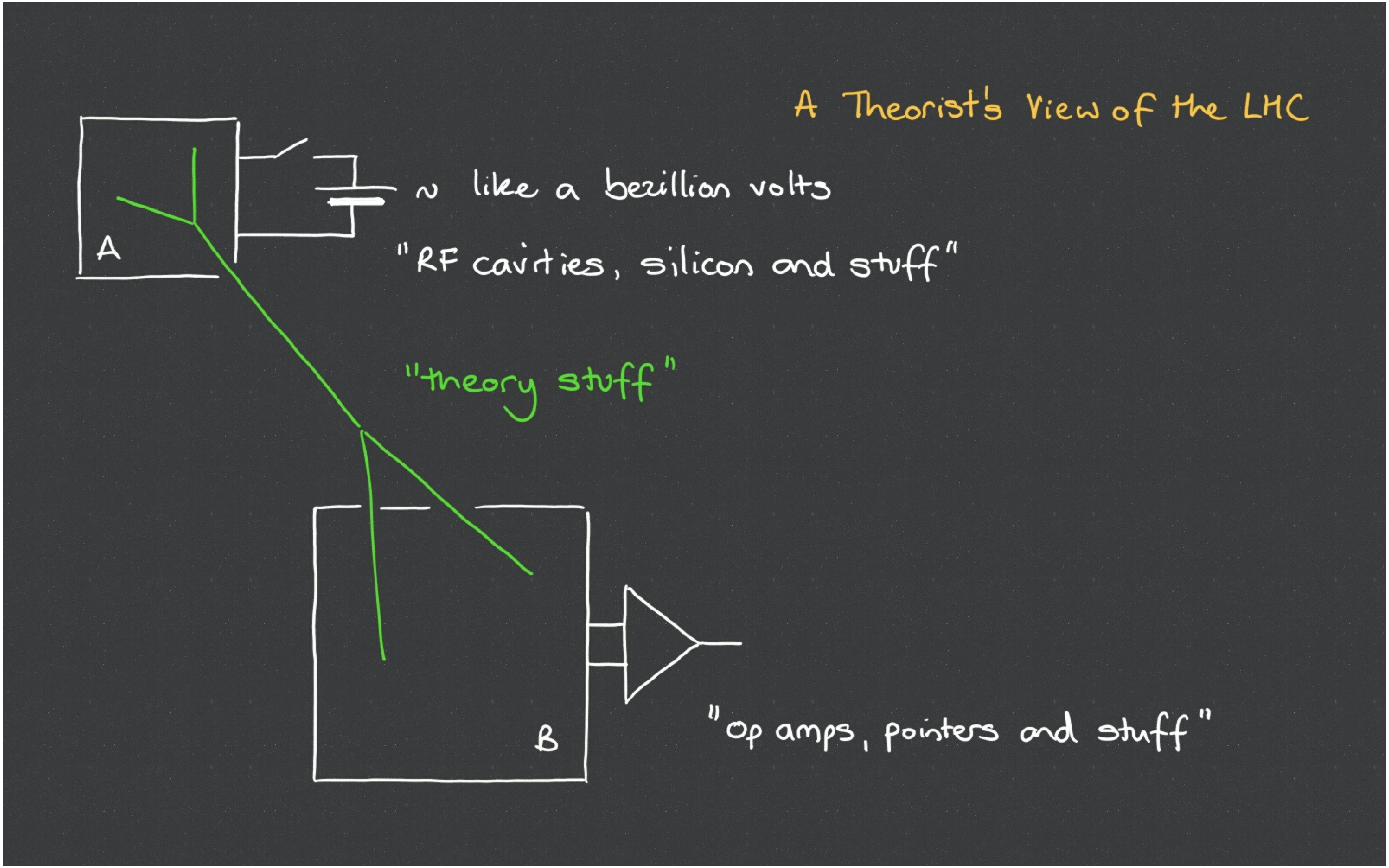
Based on :

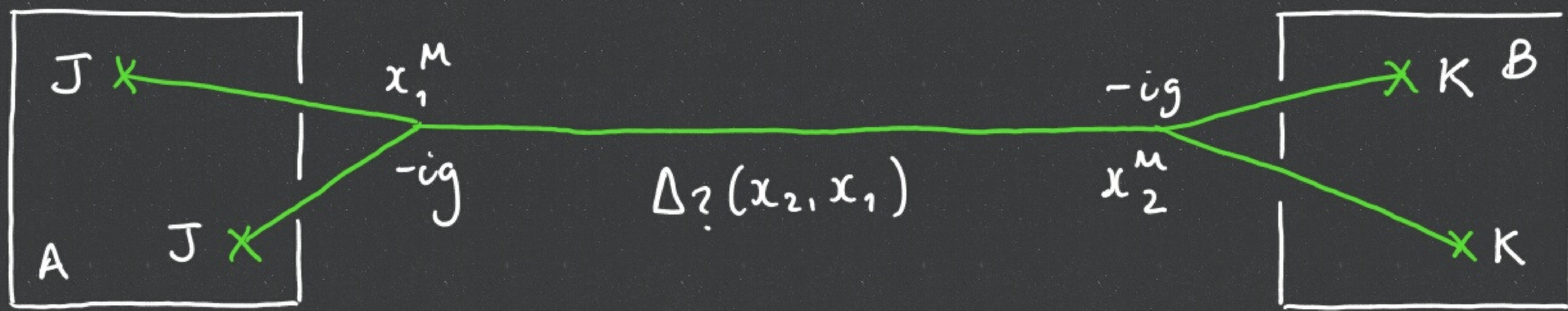
R. Dickinson, J. Forshaw, PM and B. Cox
JHEP 06 (2014) 49
arXiv: 1312.3871 [hep-th]

R. Dickinson, J. Forshaw and PM X2
in preparation

Outline

- Motivation
  - S-matrix theory and the adiabatic hypothesis
- Role of Sources
  - Modelling emission and detection in QFT
  - Manifestly-causal transition amplitudes
- Role of Negative Frequencies
  - "Neoclassical" QFT
- Conclusions

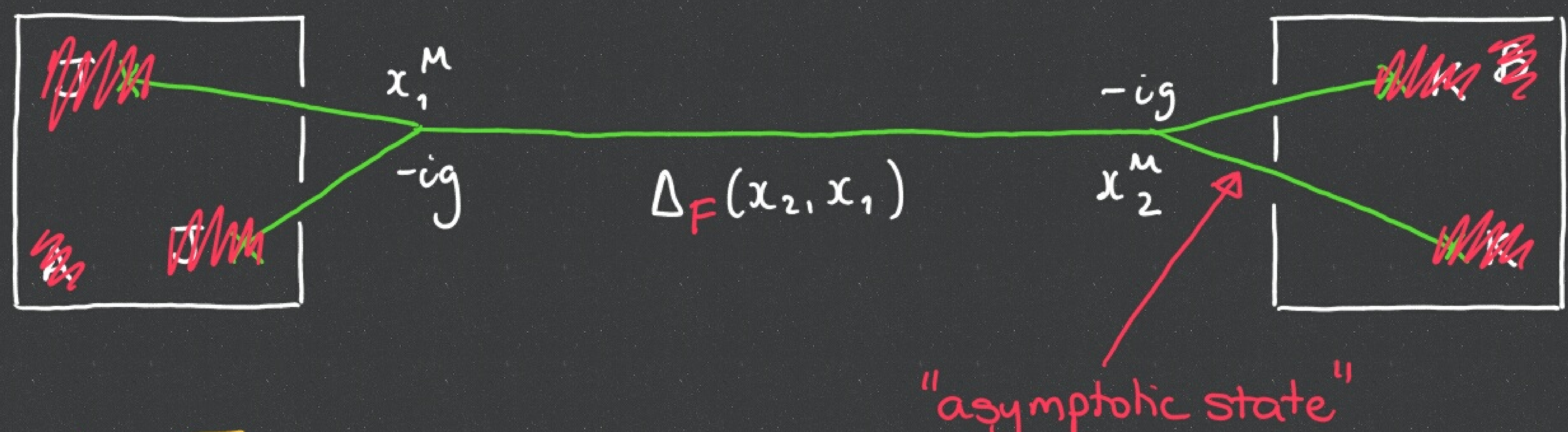

A Theorist's View of the LHC
A
~ like a bezillion volts
"RF cavities, silicon and stuff"
"theory stuff"
B
"op amps, pointers and stuff"



S-matrix Theory

1. Source J (K) acts in the infinitely-distant past (future)
2. (a) Sources act over an infinite space-time volume

   (b) Interactions turned on infinitely slowly (adiabatic hypothesis)
3. $\Delta_?(x_2, x_1)$ becomes the Feynman propagator $\Delta_F(x_2, x_1)$



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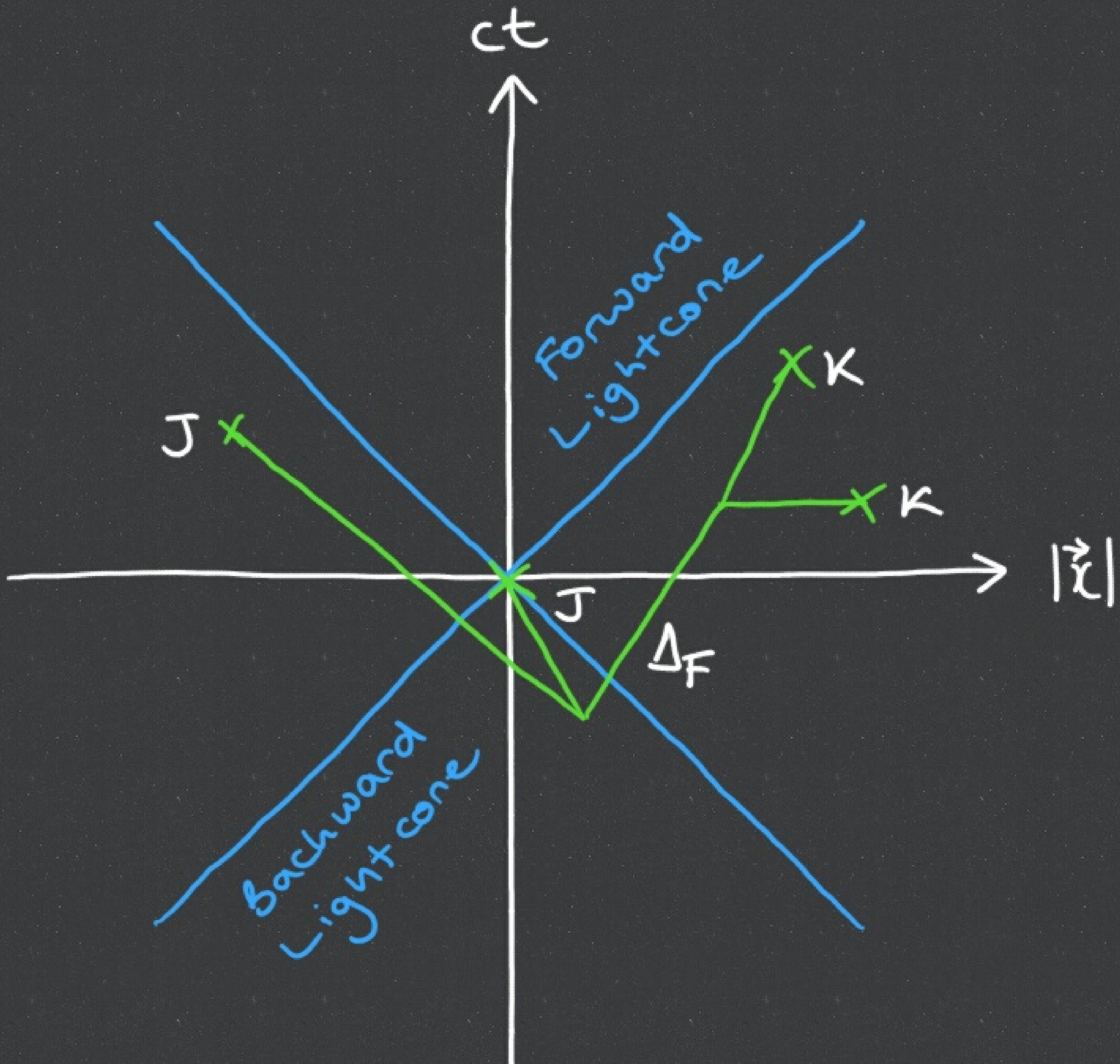
Causality

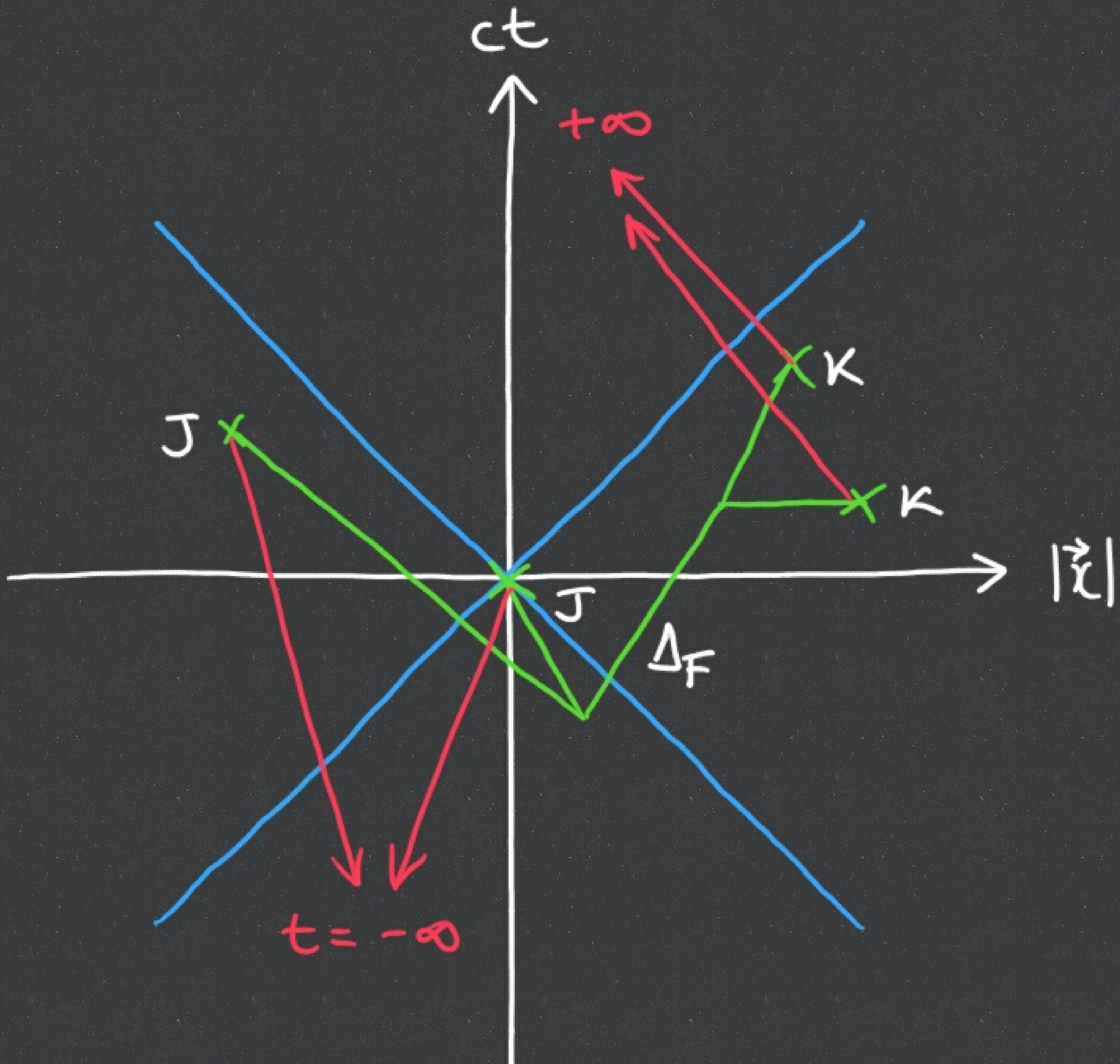


The Feynman propogator has support over all spacetime separations.

How is this in accord with the causality that we build into QFT?

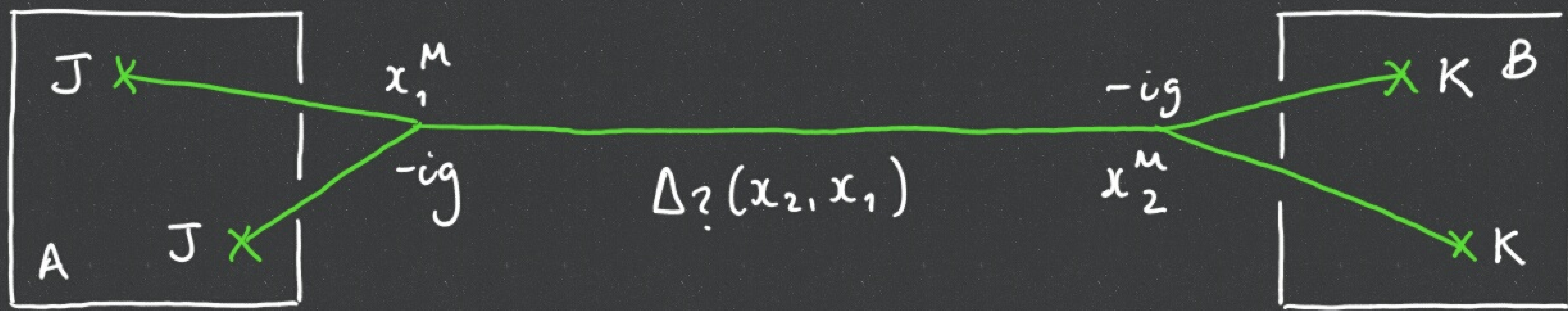
$$[\phi(t,\vec{x}),\phi(t,\vec{y})]=0$$

Causality



ALL spacetime points are in the forward lightcone of J and the backward lightcone of K.

"Causal by default"



Keeping Things Finite

1. What is $\Delta_?$ if A and B have a finite spacetime separation?

2. Surely we must get zero if no part of B lies in the forward lightcone of A?

Modelling Emission in QFT

Add a classical source $J(x)$ to the interaction Hamiltonian:

$$\mathcal{H}^{int}(x) = \frac{1}{3!} g \phi^3(x) - J(x)\phi(x)$$

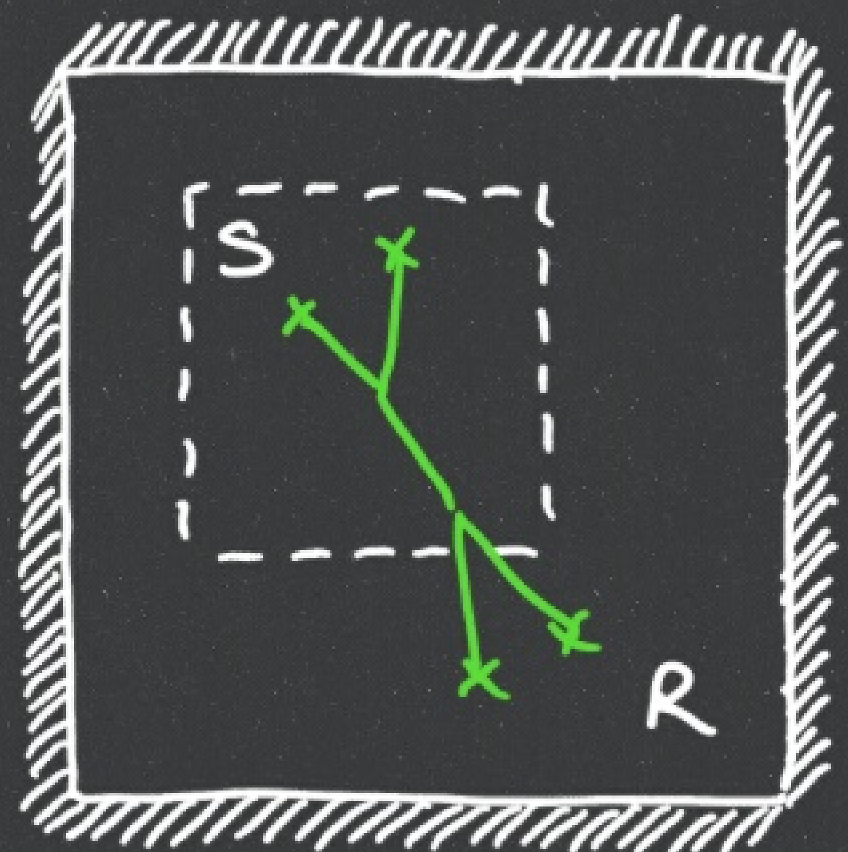
$J(x)$ acts like a driving force, i.e. the Euler-Lagrange E.o.M. is:

$$\left[-\Box - m^2 - \frac{1}{2} g \phi(x) \right] \phi(x) = J(x)$$

Production of pure states :

$$|\psi\rangle = \sum_n a_n |n\rangle$$

Modelling Detection in QFT



We divide the Universe into two systems:

S — where the emission and scatterings occur

R — which models the detector and will be "switched on" at some finite time T.

S and R should be entangled after T

⇒ mixed states, i.e. density operators

$$\rho = \sum_{n,m} a_{nm} |n\rangle\langle m|$$

What Can We Ask?

- S-matrix: we compare "in" and "out" states, i.e.

$$\langle out(+\infty) | in(-\infty)\rangle = \langle in(-\infty) | S | in(-\infty)\rangle$$

- We want to compare the

pure state before the detector acts with the

mixed state after the detector acts

$$\Rightarrow \quad P \hat{=} \langle\langle \rho_S(T) | \underbrace{\rho_S(\infty)\rangle\rangle}$$

How do we begin to write down this

complicated mixed state?

Evolution (not the Darwinian kind)

Pure States

$$|\psi(t')\rangle = \underbrace{U(t',t)}_{\text{unitary evolution operator}}|\psi(t)\rangle \qquad \langle O\rangle_t = \langle\psi(t)|O|\psi(t)\rangle$$

Mixed States

In the density-operator language, $\rho(t)$ evolves via the

von Neumann equation

$$\frac{d\rho}{dt} = -i[H,\rho] \qquad \langle O\rangle_t = \mathrm{Tr}\,\rho(t)O$$

Thermo Field Dynamics

[See e.g. Y. Takahasi and H. Umezawa, Collect. Phenom. 2 (1975) 55; F.C. Khanna et al., World Scientific (2009)]

Instead of density operators and traces, we can double up the Hilbert space and introduce "thermal vacuum states"

$$|1\rangle\rangle = \sum_n |n\rangle \otimes |n\rangle^* \qquad |\varrho(t)\rangle\rangle = \left(\rho(t)\otimes \mathbb{I}\right)|1\rangle\rangle$$

We introduce the Liouvillian: $\hat{H} \hat{=} \underbrace{H\otimes\mathbb{I}}_{H^+} - \underbrace{\mathbb{I}\otimes H^*}_{H^-}$. Then

$$|\varrho(t')\rangle\rangle = \underbrace{\hat{U}(t',t)}_{\hat{T}\exp\left[i\int_t^{t'} dx^0\, \hat{H}^{int}(x^0)\right]} |\varrho(t)\rangle\rangle \qquad \langle O\rangle_t = \langle\langle 1|O|\varrho(t)\rangle\rangle$$

Tracing out R

We are not interested in the internal dynamics of R; only its impact on how excitations move around in S.

Using the state $|1_R\rangle\rangle$ we can "trace out" R after the time T to get the mixed state of S:

$$|\rho_S(t)\rangle\rangle = \langle\langle 1_R|\rho(t)\rangle\rangle$$
$$= \langle\langle 1_R|\hat{U}(t,T)|\rho_R(T)\rangle\rangle|\rho_S(T)\rangle\rangle$$

$$|\rho_S(T)\rangle\rangle = \hat{U}_S(T,t_0)|\rho_S(t_0)\rangle\rangle$$

Source Expansion

We hide our ignorance of the interactions between S and R by writing

$$\langle\langle 1_R | \hat{U}(t,T) | \varrho_R(T) \rangle\rangle = \hat{T}\exp\left(i \int_T^t d^4x \, K[\phi^+, \phi^-] \right)$$

$$\begin{aligned} K[\phi^+, \phi^-] = {} & K(x) + K^+(x)\phi^+(x) + K^-(x)\phi^-(x) \\ & + \int_t^\infty d^4y \, K^{++}(x,y)\phi^+(x)\phi^+(y) \\ & + \int_t^\infty d^4y \, K^{+-}(x,y)\phi^+(x)\phi^-(y) \\ & + \dots \end{aligned}$$

Now ...

$$P = \langle\langle \varrho_S(T) | \varrho_S(\infty) \rangle\rangle$$

$$= \underbrace{\langle\langle \varrho_S(-\infty)|}_{\langle\langle 0|} \underbrace{\hat{U}_S^\dagger(T_1,-\infty)\, \hat{T}\exp\left(i\int_T^\infty d^4x\, K[\phi^+,\phi^-]\right)\hat{U}_S(T_1,-\infty)}_{} \underbrace{|\varrho_S(-\infty)\rangle\rangle}_{|0\rangle\rangle}$$

In order to calculate this, we need to time-order all the operators ...

Suppose x^μ is later than y^μ:

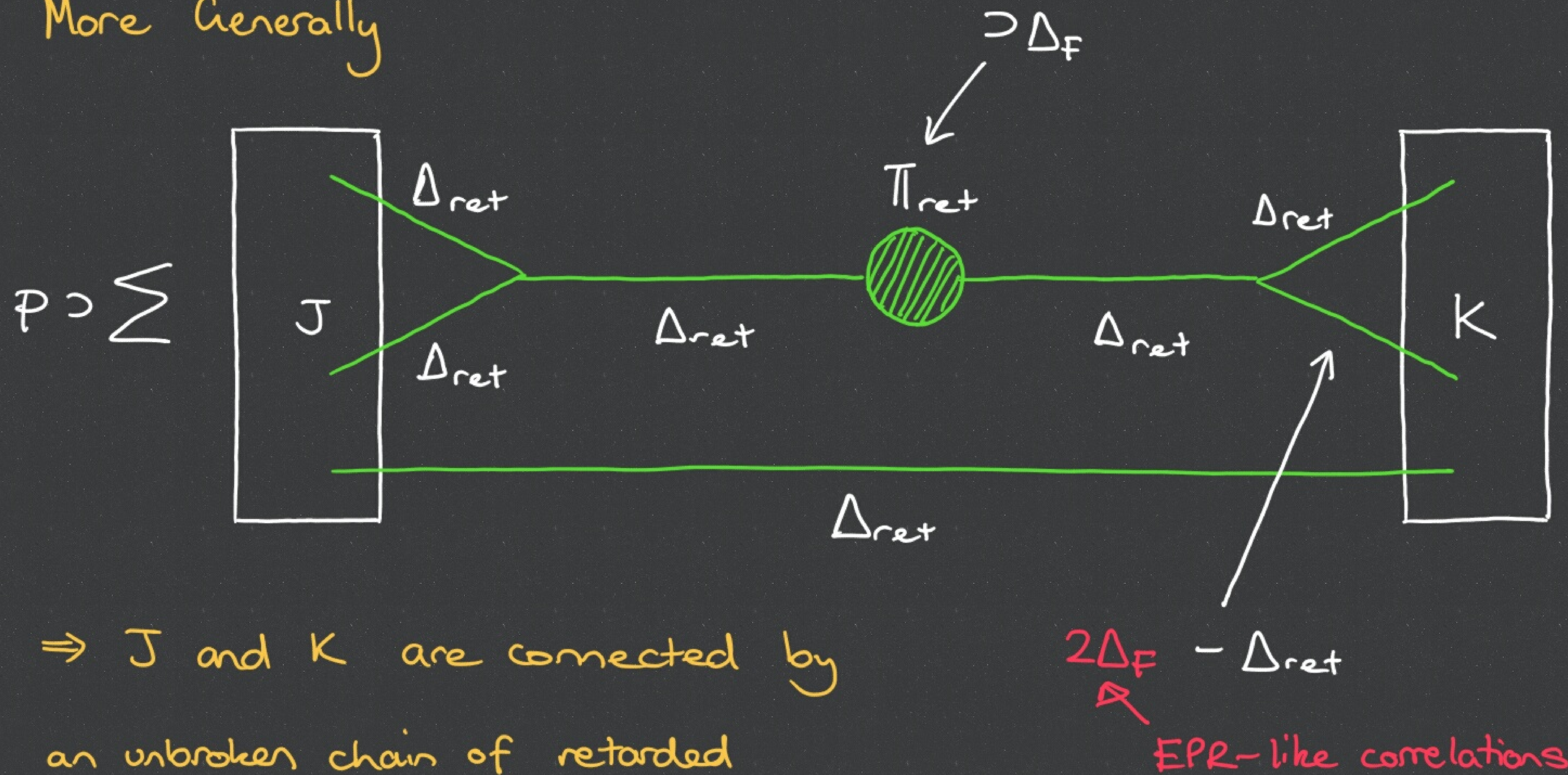
$$\phi(y)\phi(x) = \hat{T}[\phi(x)\phi(y)] - [\phi(x),\phi(y)]$$

Simple Example

$$P \supset \left| \iint_{x>y} K(x)\, J(y) \, \langle 0 | \underbrace{\big(\phi(x)\,\phi(y) - \phi(y)\,\phi(x)\big)}_{\underbrace{[\phi(x),\, \phi(y)]}_{}} | 0 \rangle \right|^2$$

$= 0$ if K is not in the forward lightcone of J.

More Generally

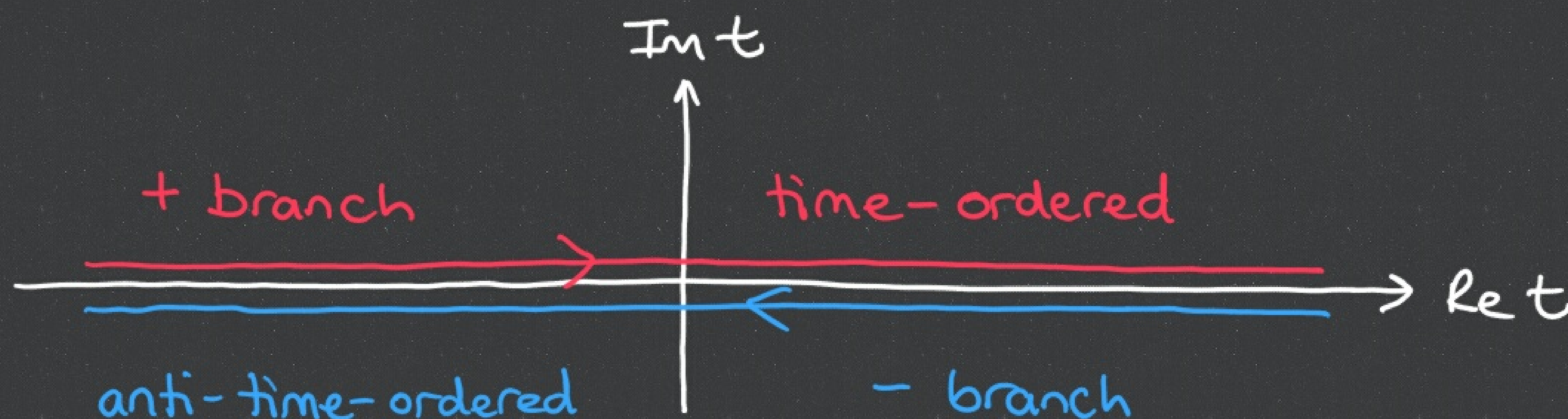


$\Rightarrow$ J and K are connected by an unbroken chain of retarded propagators.

Causal Functions

We can construct a generating functional for the Green's functions appearing in these source-to-source amplitudes using the Schwinger-Keldysh closed-time path (CTP) formalism.

⇒ path-integrals for expectation values



[See Dickinson et al. JHEP 06 (2014) 49 and references therein.]

Generating Functional

$$\Gamma_{ret}^{n\to m} = \left[\prod_{k=1}^{m} \frac{1}{i}\delta_k^+ \prod_{\ell=1}^{n} \frac{1}{i}\left(\delta_\ell^+ + \delta_\ell^-\right)\right] W[j^\pm]$$

$\delta^\pm$ are functional derivatives w.r.t. $j^\pm$.

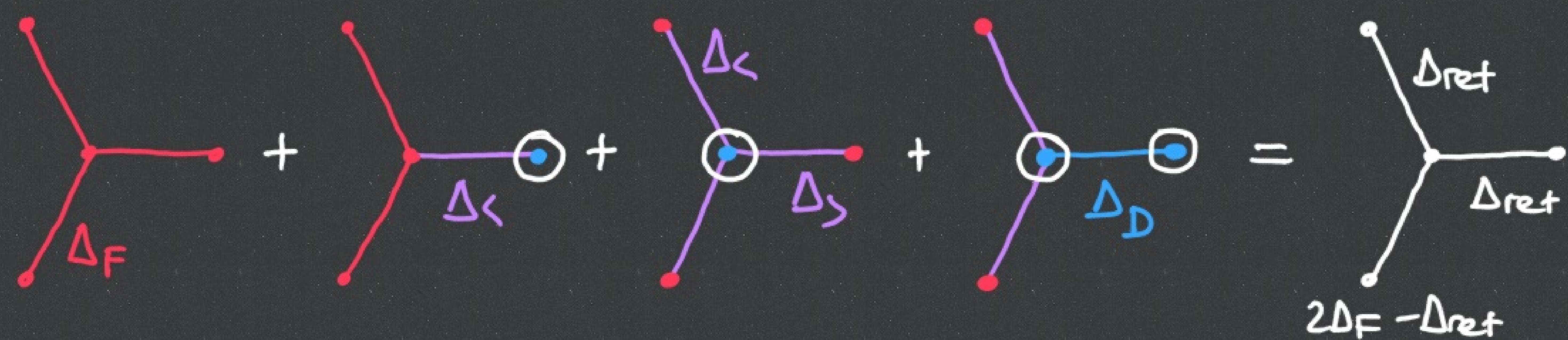
$$W[j^\pm] = \ln\left[Z_0[0] \exp\left\{\frac{g}{3!}\int_x \left[(\delta^+)^3 + (\delta^-)^3\right]\right\}\right.$$

$$\left. \times \exp\left\{-\frac{1}{2}\int_{xy}\left[\underbrace{j^+\Delta_F j^+}_{\text{Feynman}} \underbrace{- j^+\Delta_< j^- - j^-\Delta_> j^+}_{\text{Wightman}} + \underbrace{j^-\Delta_D j^-}_{\text{Dyson}}\right]\right\}\right]$$

Unitarity Cuts [R.L. Kobes and G.W. Semenoff, Nucl. Phys. B260 (1985) 714]

Summing over all $+$'s and $-$'s we have the following rule:

"Draw a given diagram and sum over all ways of circling points leaving the outgoing points uncircled."

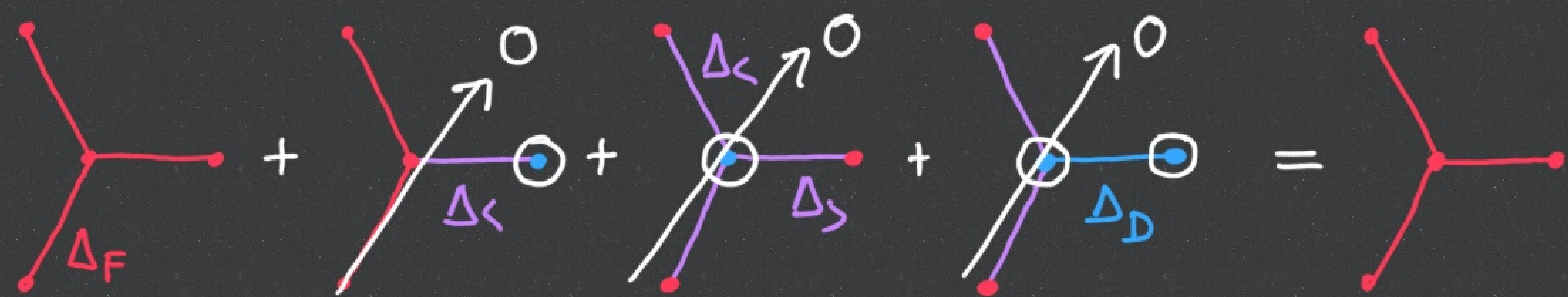


For one out-going point, these are precisely the rules that give the causal functions of the Kobes-Semenoff Unitarity Cutting Rules.

[R.L. Kobes, Phys. Rev. D43 (1991) 1269]

Positive-Frequency Limit

In the limit that the sources couple only to positive-frequency plane waves — as in the S-matrix case — only the "no circlings" contribution survives.



By virtue of unitarity, this holds to all orders in perturbation theory and we recover the usual Feynman diagram technique.

Negative - Frequencies

Feynman Propagator

$$\Delta_F(x,y) = \Theta(x^0 - y^0)\int_{LIPS} e^{-ip.(x-y)} + \Theta(y^0 - x^0)\int_{LIPS} e^{ip.(x-y)}$$

Retarded Propagator

$$\Delta_{ret}(x,y) = \underbrace{\Theta(x^0-y^0)\int_{LIPS} e^{-ip.(x-y)}}_{\text{Positive - Frequency forwards in time}} - \underbrace{\Theta(x^0-y^0)\int_{LIPS} e^{ip.(x-y)}}_{\text{Negative - Frequency forwards in time}}$$

The retarded propagator is symmetric under $E \to -E$.

Negative Energy (Warning: über provisional)

Can we make a nominal modification of QFT to permit negative-energy flow in vacuo?

We want a pair of free field operators ϕ^+ and ϕ^-, related to one another by a discrete transformation $E \to -E$.

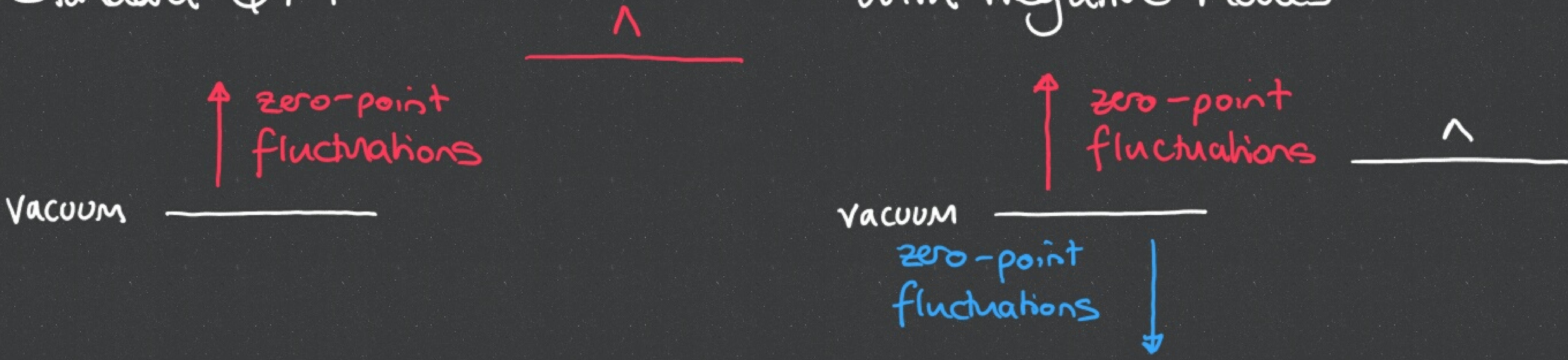
$$[a^+_p, a^{+\dagger}_q] = +(2\pi)^3 2E_p \delta^{(3)}(p-q) = \langle\langle p^+ | q^+ \rangle\rangle$$

$$[a^-_p, a^{-\dagger}_q] = -(2\pi)^3 2E_p \delta^{(3)}(p-q) = \langle\langle p^- | q^- \rangle\rangle$$

$$[a^+_p, a^{-\dagger}_q] = 0 = \langle\langle p^+ | q^- \rangle\rangle$$

Positive Bias and the Cosmological Constant Problem

Standard QFT | With Negative Modes



$$H = H^+ + H^- = \int_{\text{LIPS}} \Big(\underbrace{a_p^{+\dagger} a_p^+ + a_p^+ a_p^{+\dagger}}_{a_p^{+\dagger} a_p^+ + \cancel{(2\pi)^3 2E_p \delta^{(3)}(\vec{0})}} + \underbrace{a_p^{-\dagger} a_p^- + a_p^- a_p^{-\dagger}}_{a_p^{-\dagger} a_p^- - \cancel{(2\pi)^3 2E_p \delta^{(3)}(\vec{0})}} \Big)$$

Q's: Loop level? Condensates?

Interactions

$$\mathcal{H}^{int}(x) = \frac{g}{3!}\left\{ [\phi^+(x)]^3 + [\phi^-(x)]^3 \right\} - J(x)\left[\phi^+(x) + \phi^-(x)\right]$$

Def^n: $\tilde{\phi}^{\pm} \equiv \phi^+ \pm \phi^-$

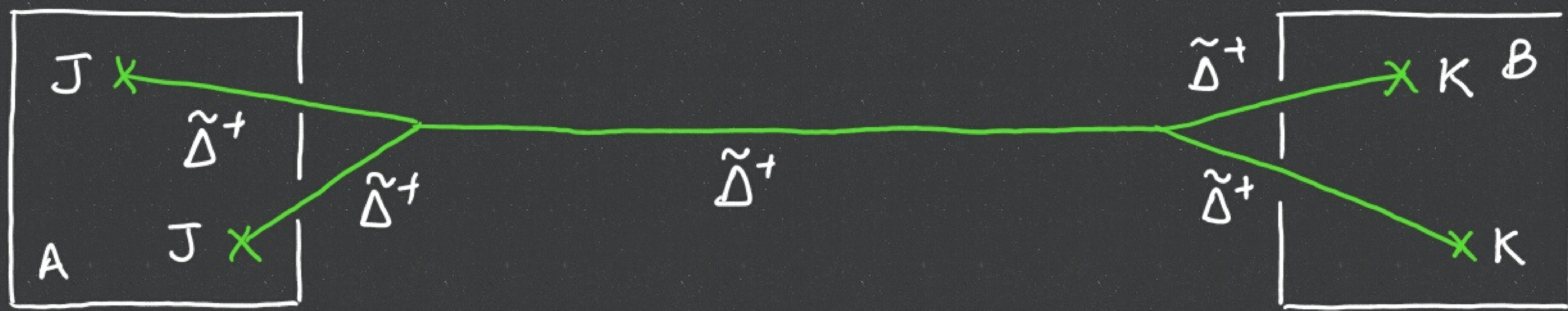
Correlation functions split into:

(i) purely causal

$$\tilde{\Delta}^+(x,y) \equiv \langle\langle T[\tilde{\phi}^{\pm}(x)\tilde{\phi}^{\pm}(y)]\rangle\rangle = \Delta_{ret}(x,y) + \Delta_{adv}(x,y)$$

(ii) purely a-causal

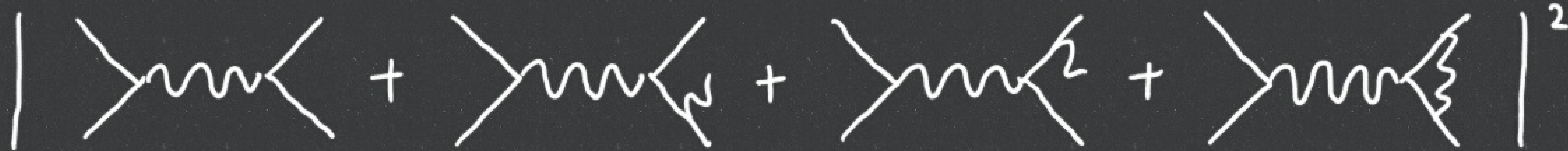
$$\tilde{\Delta}^-(x,y) \equiv \langle\langle T[\tilde{\phi}^{\pm}(x)\tilde{\phi}^{\mp}(y)]\rangle\rangle = -2\,\mathrm{Im}\,\Delta_F(x,y)$$



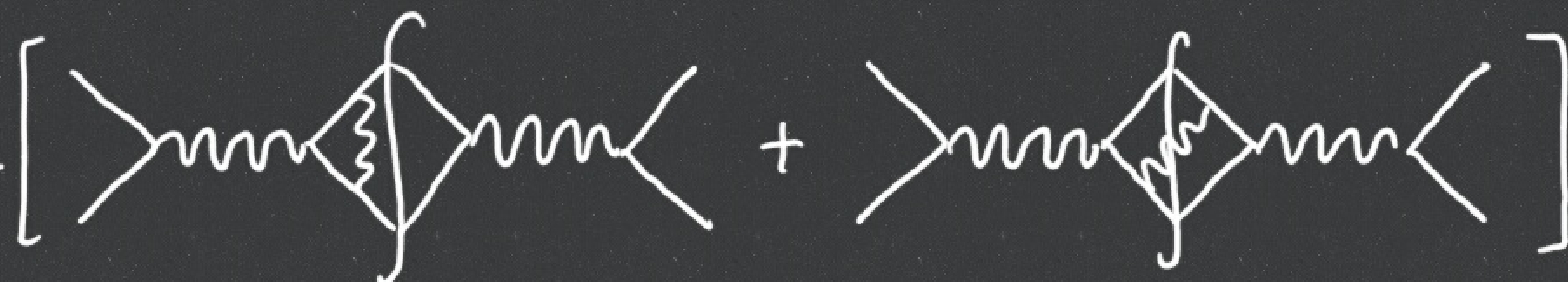
Tree-level source-to-source amplitudes are completely causal (when scattering plane waves.)

Problems: Bloch–Nordsieck Cancellation

In QED, we need the interference of real and virtual emissions for IR divergences to cancel, i.e.



$\supset 2\,\mathrm{Re}$



virtual real

But our matrix elements are either purely real or purely imaginary: $|M_{tree} + M_{real} + M_{virt}|^2 = |M_{tree} + M_{real}|^2 + |M_{virt}|^2$ ✗

Loop Corrections

The one-particle irreducible effective action

$$V(\tilde{\varphi}^{\pm}) \supset \frac{1}{4}\Big\{\underbrace{m^2\left[(\tilde{\varphi}^+)^2+(\tilde{\varphi}^-)^2\right]}_{\text{leading UV divergence has cancelled}} + ig^2\tilde{\varphi}^+\tilde{\varphi}^-\underbrace{\int_k\left(\frac{1}{k^2-M^2+i\epsilon}\right)^2}_{\text{UV divergent mixing}}$$

⇒ potential suppression of UV sensitivity?

φ: naturalness?

Electroweak Oblique Corrections

Standard matrix elements for neutral and charge current reactions

$$\mathcal{M}_{NC} = \frac{e^2 Q Q'}{q^2 + \Pi_A} + \frac{e^2}{s^2 c^2} \frac{(I_3 - s_*^2 Q)(I_3' - s_*^2 Q')}{q^2 - m_Z^2 + \Pi_Z + \frac{\Pi_{ZA}^2}{q^2 + \Pi_A}}$$

$$\mathcal{M}_{CC} = \frac{e^2}{2s^2} \frac{I_+ I_-}{q^2 - m_W^2 + \Pi_W}$$

$$s_*^2 \equiv s^2 + sc \frac{\Pi_{ZA}}{q^2 + \Pi_A} \; ; \quad s \equiv \sin\theta_W \; ; \quad c \equiv \cos\theta_W$$

[See M. E. Peskin and T. Takeuchi, Phys. Rev. D 46 (1992) 381]

Adding in the Negative Frequency Modes ...

For instance

$$\mathcal{M}_{cc} = \frac{e^2}{2s^2} I_+ I_- \frac{2(q^2 - m_W^2 + i\,\mathrm{Im}\,\Pi_W)}{(q^2 - m_W^2 + i\,\mathrm{Im}\,\Pi_W)^2 - (\mathrm{Re}\,\Pi_W)^2}$$

It would be a miracle if this gave the standard result, but let's try anyway...

Renormalized Quantities

(i) Pole mass m_W:

$$\left[(q^2 - M_W^2 + i\,\mathrm{Im}\,\Pi_W)^2 - (\mathrm{Re}\,\Pi_W)^2 \right]_{q^2 = \bar{M}_W^2} = 0$$

$\Rightarrow$ standard gap equation

$$\bar{m}_W^2 = m_W^2 - \Pi_W(\bar{m}_W^2) \quad \checkmark$$

Renormalized Quantities

(ii) Wavefunction renormalization

$$Z_W^{-1} = \frac{d}{dq^2} \frac{(q^2 - m^2 + i\,\mathrm{Im}\,\Pi_W)^2 - (\mathrm{Re}\,\Pi_W)^2}{2(q^2 - M_W^2 + i\,\mathrm{Im}\,\Pi_W)} \Bigg|_{q^2 = \bar{M}_W^2}$$

$$= 1 + \frac{d\Pi_W}{dq^2}\Bigg|_{q^2 = \bar{M}_W^2} \quad \checkmark$$

$$\therefore \mathcal{M}_{CC} = \frac{e^2}{2s^2} I_+ I_- \frac{Z_W}{q^2 - \bar{M}_W^2} \quad \checkmark$$

Surprise!

The neutral current piece works too and we obtain the standard running parameters Z_{W*}, Z_{Z*}, $\bar{m}_{W*}$, $\bar{m}_{Z*}$, s_*, e_*

$\Rightarrow$ standard parametrization of the EW S, T, U parameters.

How has this worked out?

Potentially: (i) resonance phenomena?

(ii) breakdown of naive perturbation theory?

$\Leftarrow$ degenerate spectrum of "particles" mixed by their interaction with the classical source J.

Conclusions

- S-matrix theory is causal by default.
- If we want manifest causality, we need to keep track of sources and sinks and finite times.
- Obtained manifestly-causal transition amplitudes that are consistent with S-matrix results. (x3 ways)
- Observed the relevance of negative-energy flow to causality.
- Played with a nominal departure from standard QFT that permits negative-energy states (with intriguing results).

Backup Slides

A Mathematical Excursion: Bicomplex Numbers

Complex numbers $\mathbb{C}$: the real numbers $\mathbb{R}$

+ an imaginary unit "i": $i^2 = -1$, $i^* = -i$

Bi-complex numbers $\mathbb{T}$: the complex numbers $\mathbb{C}$

+ a second imaginary unit "j": $j^2 = -1$, $j^{\star} = -j$.

$\Rightarrow$ two "complex" conjugations

$i^{\star} = i$, $j^* = j$

Def^n: $k \hat{=} ij$: $k^2 = +1$, $k^* = -k$, $k^{\star} = -k$, $k^{*\star} = k$

$(*\star)$ defines the $\mathbb{T}$-norm, which is positive semi-definite.

Free Scalar Field

$$\phi^{+}(x) = +\int_{LIPS} \left(a^{+}_{p}\, e^{-ip\cdot x} + a^{+\dagger}_{p}\, e^{ip\cdot x}\right)$$

$$\phi^{-}(x) = -\int_{LIPS} \left(a^{-}_{p}\, e^{ip\cdot x} + a^{-\dagger}_{p}\, e^{ip\cdot x}\right)$$

with all the usual properties

$$[\phi^{\pm}(x), \phi^{\pm}(y)]\Big|_{x^0=y^0} = 0$$

$$[\phi^{+}(x), \phi^{-}(y)] = 0.$$

Dressed Propagators

$$\tilde{\Delta}^{+}(p^2) = \frac{2i(p^2 - M^2 + i\,\mathrm{Im}\,\Pi)}{(p^2 - M^2 + i\,\mathrm{Im}\,\Pi)^2 - (\mathrm{Re}\,\Pi)^2}$$

$$\tilde{\Delta}^{-}(p^2) = \frac{-2i\,\mathrm{Re}\,\Pi}{(p^2 - M^2 + i\,\mathrm{Im}\,\Pi)^2 - (\mathrm{Re}\,\Pi)^2}$$

Compare with

$$\Delta_F(p^2) = \frac{1}{p^2 - M^2 + \Pi}$$