

4d/5d branes from special geometry

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University of Liverpool, April 2015

Based on: [arXiv 1501.07863](https://arxiv.org/abs/1501.07863) (with D.Errington and T.Mohaupt)
and work to appear

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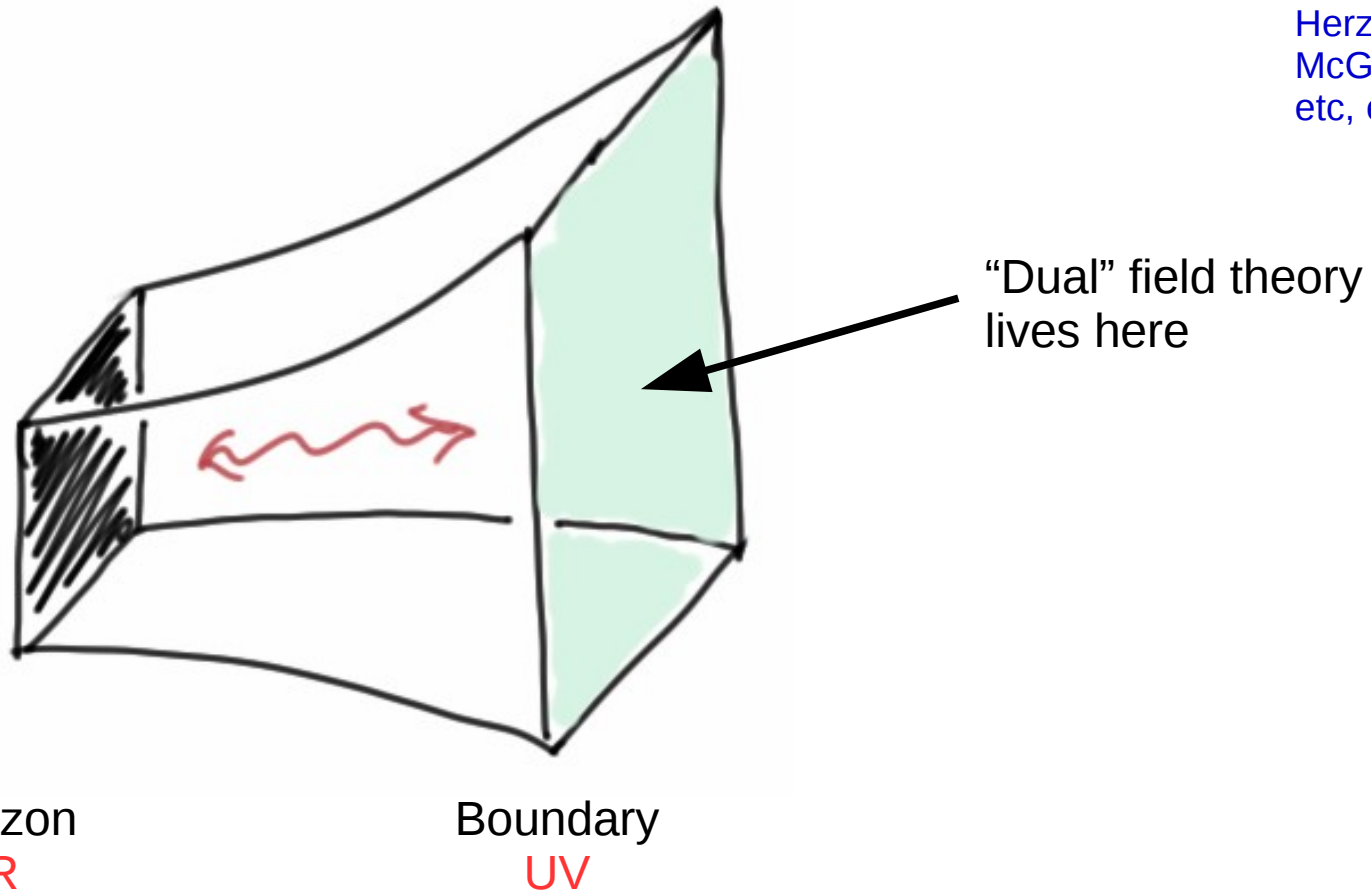
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I. Introduction to AdS/CMT

AdS/CMT basics

Holographic thinking leads to remarkable insight into strong-coupling limit of condensed matter systems by studying classical gravity duals

Hartnoll (2009, 2011)
Herzog (2009)
McGreevy (2009)
etc, etc, etc.



Propagation in bulk dual to RG flow in boundary theory

QFT at finite temperature T
and entropy (density) s



Black brane solution in the bulk
with temperature $T_H = T$
and entropy (density) s

Witten (1998)

Approaches to the duality

Top-down

SUGRA with UV completion



Investigate dual field theories

Pros:

Analytical results
Stringy embedding

Cons:

Not sure what you'll end up with!

Bottom-up

Construct a gravity dual



Field theory under investigation

Pros:

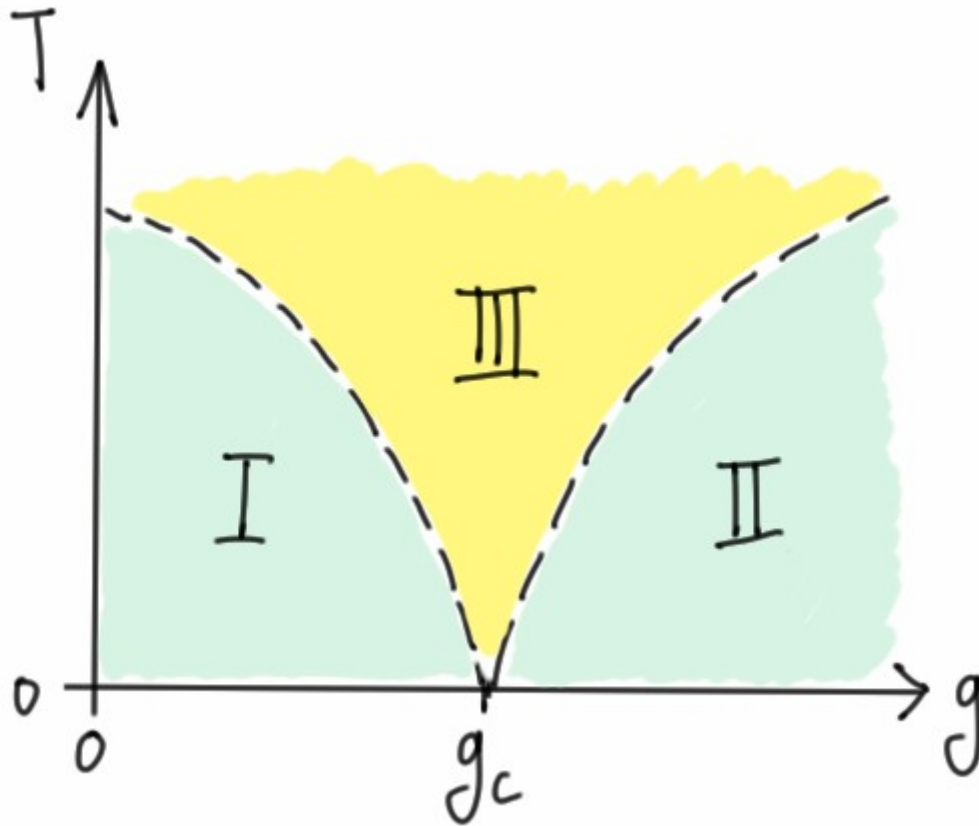
Field theory guaranteed “desirable” properties

Cons:

Often numerical

How to find a string embedding/UV completion?

An application: quantum criticality



Quantum phase transition (QPT)
A macroscopic rearrangement of the ground state of a system as some external parameter is varied.

Quantum Critical Region
Finite temperature region where the system can be described by excitations of the scale/conformal invariant ground state at the QCP.

Example

5d EM-CS theory with non-zero charge density and external magnetic field

I: $s \sim \sqrt{B_c - B}$

II: $s \sim \frac{T}{B - B_c}$ cf. Fermi liquid

III: $s \sim T^{1/3}$

D'Hoker and Kraus (2009-12)

Quantum critical behaviour \longrightarrow (1+1)-dim dual with $z = 3$ and relevant operator with $\Delta = 2$

Nernst branes

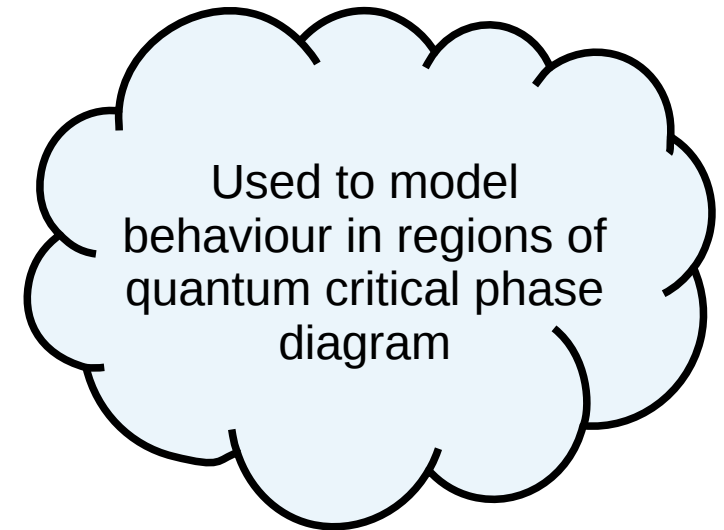
Nernst branes: gravity solutions with zero entropy density at zero temperature

Contrast to simplest charged (RN) black brane, with non-zero entropy at zero temperature

Third Law of Thermodynamics (Planckian version)
Entropy (density) goes to zero at zero temperature, with all other quantities held fixed.



Existence of unique ground state in the field theory



Investigated in EMD and EM-CS theories with both electric and magnetic charges

[D'Hoker, Kraus \(2009\)](#)

[Goldstein, Kachru, Prakash, Trivedi \(2009\)](#)

[Goldstein, Iizuka, Kachru, Prakash, Trivedi \(2010\)](#)

Extremal “Nernst branes” in N=2 supergravity

[Barisch, Lopes Cardoso, Haack, Nampuri, Obers \(2011\)](#)

[Barisch-Dick, Lopes Cardoso, Haack, Nampuri \(2012\)](#)

[Goldstein, Obers, Véliz-Osorio \(2014\)](#)

Nernst branes have finite curvature invariants at horizon, unlike “small black holes”

Aims of the talk

1. Construct non-extremal Nernst branes in 4d $N=2$ gSUGRA
2. Discuss holography in terms of hvLif theories
3. Construct non-extremal Nernst branes in 5d $N=2$ gSUGRA
4. Understand relationship between 4d and 5d geometries

II. Four-dimensional Nernst branes from the real formulation of special geometry

d=4, N=2 gSUGRA

Focus on $\mathcal{N} = 2$ Fayet-Iliopoulos (FI) $U(1)$ gauged supergravity in four dimensions

Field content

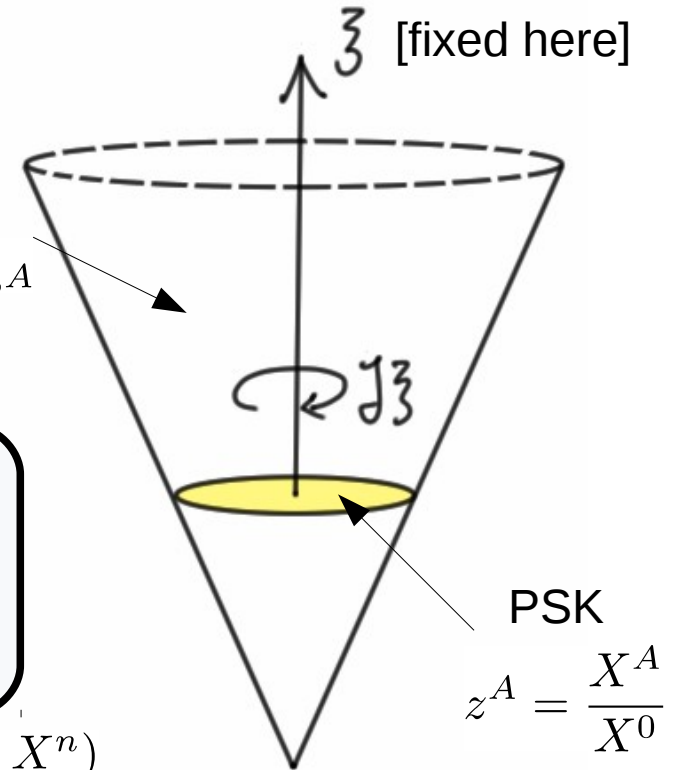
Gravity multiplet $(g_{\hat{\mu}\hat{\nu}}, A_{\hat{\mu}}^0 | \dots)$

Vector multiplets $(A_{\hat{\mu}}^A, z^A | \dots)_{A=1, \dots, n}$

CASK
 $(X^I)_{I=0, A}$

Lagrangian

$$e_4^{-1} \mathcal{L}_4 = -\frac{1}{2} R_4 - g_{IJ} \partial_{\hat{\mu}} X^I \partial^{\hat{\mu}} X^J + \frac{1}{4} \mathcal{I}_{IJ} F_{\hat{\mu}\hat{\nu}}^I F^{J|\hat{\mu}\hat{\nu}} + \frac{1}{4} \mathcal{R}_{IJ} F_{\hat{\mu}\hat{\nu}}^I \tilde{F}^{J|\hat{\mu}\hat{\nu}} - V(X, \bar{X})$$



Couplings determined by **prepotential** $F(X) = \frac{f(X^1, \dots, X^n)}{X^0}$

Scalar potential

$$V(X, \bar{X}) = N^{IJ} \partial_I W \bar{\partial}_J \bar{W} - 2|W|^2$$

$$W = 2(g^I F_I - g_I X^I)$$

(g^I, g_I) FI parameters



Magnetic/electric fluxes if coming from flux compactifications

Outline

Look for stationary field configurations $\implies \exists$ timelike isometry

Time-like KK reduction to Euclidean theory

Breitenlohner, Maison, Gibbons (1988)
Cortés, Mohaupt *et al* (2004-09)

Rewrite Euclidean action using real formulation of special geometry

Freed (1999)
Alekseevsky, Cortés, Devchand (1999)
Mohaupt and Vaughan (2011)

Solve equations of motion \longrightarrow Instanton solution

Lift instanton and impose regularity on 4d solitonic solution

Used to construct solutions in (un)gauged supergravity

Mohaupt and Vaughan (2011)
Klemm and Vaughan (2012)
Gnecchi, Hristov, Klemm, Toldo, Vaughan (2013)
PD, Mohaupt (2013)
Errington, Mohaupt, Vaughan (2014)

Dimensional reduction

Reduction ansatz (simple case)

Static metric $ds_4^2 = -e^\phi dt^2 + e^{-\phi} ds_3^2$

Single electric charge $A^0 = 2\hat{q}^0 dt$

“Axion-free” configuration $\text{Re}(z^A) = 0$

Only keep electric fluxes g_1, \dots, g_n

Coming soon! Dyonic branes

Real formulation of special geometry [Mohaupt and Vaughan \(2011\)](#)

$$Y^I = e^{\phi/2} X^I$$

Define a real symplectic vector $(q^a)_{a=0, \dots, 2n+1} = \text{Re}(Y^I, F_I(Y))$

$$F(X) \xrightarrow{\text{Legendre transform}} H(q) \quad \text{Hesse potential}$$

Useful to define $q_a = -\tilde{H}_{ab} q^b$ where $\tilde{H}_{ab} = \partial_a \partial_b \tilde{H}$ for $\tilde{H}(q) = -\frac{1}{2} \log(-2H)$

Recover metric dof through $e^\phi = -2H(q) = \frac{1}{2} (-q_0 f(q_1, \dots, q_n))^{-1/2}$

Axion-free condition sets $(q^a) = (x^0, 0, \dots, 0; 0, y_1, \dots, y_n)$

Euclidean theory

Three-dimensional (Euclidean) Lagrangian

$$e^{-1} \mathcal{L}_3 = -\frac{1}{2} R_3 - \tilde{H}_{ab} (\partial_\mu q^a \partial^\mu q^b - \partial_\mu \hat{q}^a \partial^\mu \hat{q}^b - g^a g^b) + 4(g^a q_a)^2$$

Full Lagrangian in 3d described by (gauged) NLSM with para-QK target manifold

Cortés, PD, Mohaupt, Vaughan (to appear)

Equations of motion

$$\nabla^2 \hat{q}_a = 0$$

$$\nabla^2 q_a + \frac{1}{2} \partial_a \tilde{H}^{bc} (\partial_\mu q_b \partial^\mu q_c - \partial_\mu \hat{q}_b \partial^\mu \hat{q}_c) - \frac{1}{2} \partial_a \tilde{H}_{bc} g^b g^c + 4 \tilde{H}_{ab} g^b (g^c q_c) = 0$$

$$-\frac{1}{2} R_{3|\mu\nu} - \tilde{H}^{ab} (\partial_\mu q_a \partial_\nu q_b - \partial_\mu \hat{q}_a \partial_\nu \hat{q}_b) + g_{\mu\nu} \left(-\tilde{H}_{ab} + 4q_a q_b \right) g^a g^b = 0$$

We will solve the **full** equations of motion, i.e. including “backreaction”

The black brane solution

We find the following solution:

PD, Errington, Mohaupt (2015)

Line element $ds_{(4)}^2 = -e^\phi dt^2 + e^{-\phi+4\psi} d\tau^2 + e^{-\phi+2\psi} (dx^2 + dy^2)$

$$e^\phi = \frac{1}{2} (-q_0 f(q_1, \dots, q_n))^{-\frac{1}{2}} \quad e^{-4\psi} = \left(\frac{\sinh(B_0\tau)}{B_0} \right)^3 e^{B_0\tau}$$

$$q_A(\tau) = \pm \frac{3}{8ng_A} e^{\frac{1}{2}B_0\tau} \left(\frac{\sinh(B_0\tau)}{B_0} \right)^{\frac{1}{2}} \quad q_0(\tau) = \pm -\frac{Q_0}{B_0} \sinh \left(B_0\tau + B_0 \frac{h_0}{Q_0} \right)$$

Physical scalars

$$z^A = -i \left(\frac{-q_0 q_A^2}{f(q_1, \dots, q_n)} \right)^{\frac{1}{2}}$$

Gauge field $A_t^0(\tau) = \frac{1}{2} \left(\frac{B_0}{Q_0} \right) \left[\coth \left(B_0\tau + \frac{B_0 h_0}{Q_0} \right) - 1 \right] \quad A_t(\infty) = 0$

Two parameter family of solutions depending on (B_0, h_0)

In the limit $B_0 \rightarrow 0$ we recover the extremal Nernst branes of [Barisch et al.](#)

A coordinate redefinition

Introduce radial coordinate ρ

$$e^{-2B_0\tau} = 1 - \frac{2B_0}{\rho} \equiv W(\rho)$$

Horizon

$$\rho = 2B_0$$

$$\tau \rightarrow \infty$$

Boundary

$$\rho \rightarrow \infty$$

$$\tau = 0$$

In terms of this we have

$$q_0 = \pm \frac{\mathcal{H}_0}{W^{\frac{1}{2}}} \quad \mathcal{H}_0(\rho) = - \left[\frac{Q_0}{B_0} \sinh \left(\frac{B_0 h_0}{Q_0} \right) + \frac{Q_0 e^{-\frac{B_0 h_0}{Q_0}}}{\rho} \right]$$

$$q_A = \pm \frac{3}{8ng_A} (\rho W)^{-\frac{1}{2}}$$

Line element

$$ds_{(4)}^2 = -\mathcal{H}^{-\frac{1}{2}} W \rho^{\frac{3}{4}} dt^2 + \mathcal{H}^{\frac{1}{2}} \rho^{-\frac{7}{4}} \frac{d\rho^2}{W} + \mathcal{H}^{\frac{1}{2}} \rho^{\frac{3}{4}} (dx^2 + dy^2)$$

$$\mathcal{H}(\rho) \equiv \pm 4 \left(\frac{3}{8n} \right)^3 f \left(\frac{1}{g_1}, \dots, \frac{1}{g_n} \right) \mathcal{H}_0(\rho)$$

Line element is that of extremal Nernst brane dressed with “blackening factor”

Thermodynamics

Holographic dictionary: thermodynamics in bulk \longleftrightarrow thermodynamics on boundary

We can compute bulk thermodynamic quantities directly from metric and gauge fields

Entropy density

$$s = (ZQ_0)^{\frac{1}{2}} (2B_0)^{\frac{1}{4}} e^{\frac{B_0 h_0}{2Q_0}} \quad Z \equiv \pm 4 \left(\frac{3}{8n} \right)^3 f \left(\frac{1}{g_1}, \dots, \frac{1}{g_n} \right)$$

Temperature

Hawking temperature obtained from near-horizon metric by regularising Euclidean time circle

$$4\pi T_H = (ZQ_0)^{-\frac{1}{2}} (2B_0)^{\frac{3}{4}} e^{-\frac{B_0 h_0}{2Q_0}}$$

Chemical potential

Chemical potential given by asymptotic value of t-component of gauge field

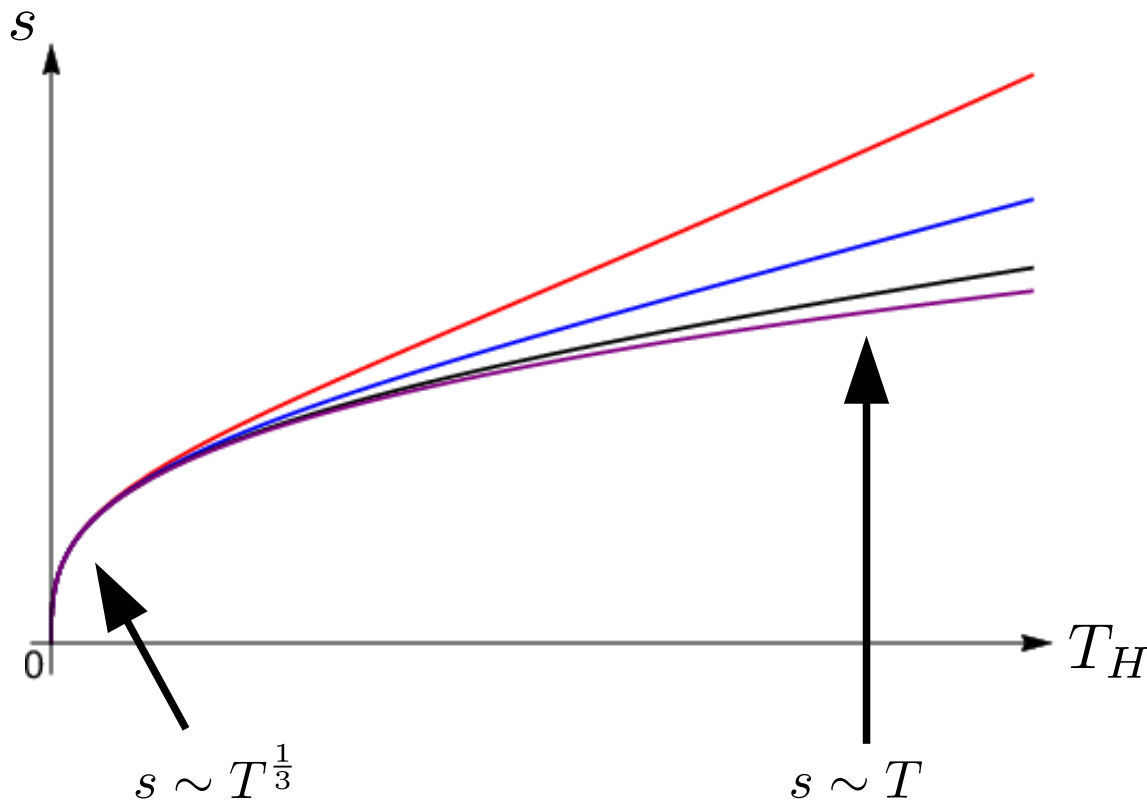
e.g. Hartnoll (2009)

$$\mu \equiv A_t^0(\tau = 0) = \frac{1}{2} \left(\frac{B_0}{Q_0} \right) \left[\coth \left(\frac{B_0 h_0}{Q_0} \right) - 1 \right]$$

Thermodynamics

Equation of state

$$s^3 = 4\pi(ZQ_0)^2 T_H \left(1 + \frac{2\pi s T_H}{Q_0 \mu} \right)$$



For fixed Q_0, μ, Z
 $s \rightarrow 0$ as $T_H \rightarrow 0$

↓
Nernst law

Figure: Plot of equation of state for fixed Q_0, μ, Z showing smooth crossover behaviour

HvLif holography

Solutions *not* asymptotically AdS_4 \longrightarrow Usual dictionary doesn't apply!

Hyperscaling-violating Lifshitz holography

Huijse, Sachdev, Swingle (2011)

Dong, Harrison, Kachru, Torroba, Wang (2012)

Perlmutter (2012)

Consider metrics of the form

$$ds_{d+2}^2 = r^{-\frac{2(d-\theta)}{d}} \left(-r^{-2(z-1)} dt^2 + dr^2 + dx_i dx_i \right) \quad z = 1, \theta = 0 \text{ metric on } \text{AdS}_{d+2}$$

Scale transformations $x_i \mapsto \lambda x_i$ $t \mapsto \lambda^z t$ $r \mapsto \lambda r$ $ds \mapsto \lambda^{\frac{\theta}{d}} ds$

In boundary field theory

z dynamical critical (Lifshitz) exponent

θ hyperscaling-violating exponent

Huijse, Sachdev, Swingle (2011)

Null-energy condition (NEC) imposes constraints on allowed values of (z, θ) :

$$(d - \theta)(d(z - 1) - \theta) \geq 0, \quad (z - 1)(d + z - \theta) \geq 0$$

Dong, Harrison, Kachru, Torroba, Wang (2012)

Scaling arguments in field theory imply entropy density behaves as $s \sim T^{\frac{d-\theta}{z}}$

Huijse, Sachdev, Swingle (2011)
Sachdev (2012)

HvLif holography

Start with $h_0 = 0$ which corresponds to infinite chemical potential

$$\underline{T = 0}$$

Metric is *globally* hvLif with $(z, \theta) = (3, 1)$

Conjecture

This solution is dual to the **ground state** of a (2+1)-dimensional QFT with $(z, \theta) = (3, 1)$

Solution has similar behaviour to some domain walls in gSUGRA, which are taken as ground states in absence of more symmetric solutions

Mayer, Mohaupt (2004)

$$\underline{T \neq 0}$$

Metric interpolates between near-horizon Rindler and asymptotic hvLif with $(z, \theta) = (3, 1)$

Conjecture

This solution is dual to a **thermal state** of the (2+1)-dimensional QFT with $(z, \theta) = (3, 1)$

$$\text{Gravity solution} \quad s = (4\pi Z^2 Q_0^2 T_H)^{\frac{1}{3}} \quad \longleftrightarrow \quad \text{Field theory} \quad s \sim T^{\frac{(2-1)}{3}} = T^{\frac{1}{3}}$$

$\theta = d - 1$ describes compressible states with hidden Fermi surfaces

Huijse, Sachdev, Swingle (2011)

Flow between hvLif theories

Now turn on finite chemical potential ($h_0 \neq 0$)

$$\underline{T = 0}$$

Solution interpolates between different hvLif geometries



Conjecture

This solution is dual to an RG flow between two (2+1)-dimensional QFTs: one with $(z, \theta) = (3, 1)$ in the IR, and one with $(z, \theta) = (1, -1)$ in the UV

Smooth gravity solution \longrightarrow UV and IR 'phases' related by smooth crossover?

$$\underline{T \neq 0}$$

Interpolates between near-horizon Rindler and asymptotic hvLif with $(z, \theta) = (1, -1)$

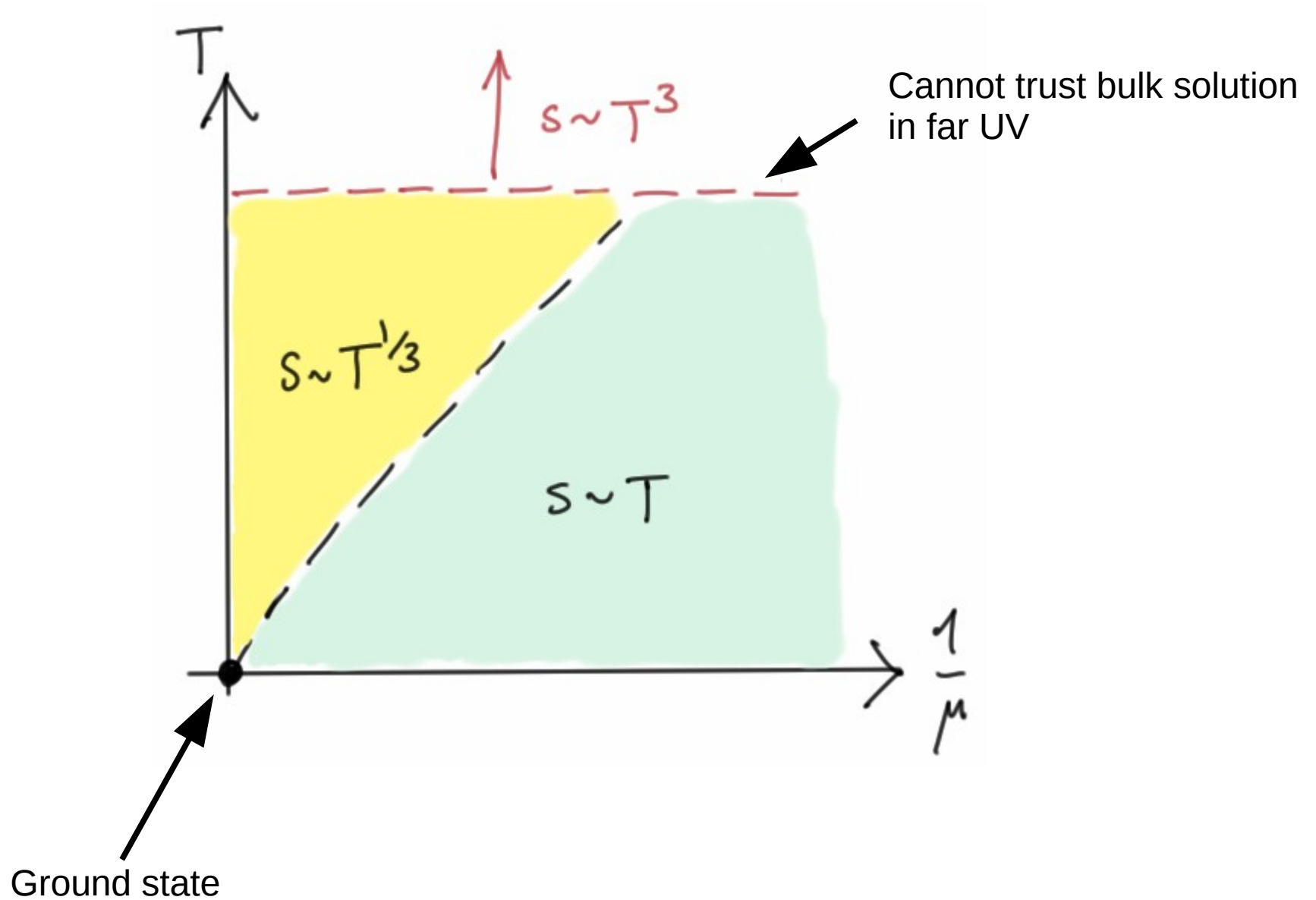
Gravity solution has $s \sim T^{\frac{1}{3}}$ for low temperatures \longrightarrow Expected for IR theory!

For high temperatures (UV physics) gravity solution gives $s \sim T$

BUT field theory would give $s \sim T^{\frac{(2+1)}{1}} = T^3 \longrightarrow$ additional UV dofs?

Finite temperature crossover between hvLif theories investigated recently in EMD theories

Phase diagram



III. Lift to five dimensions

Lifting to five dimensions

For $h_0 \neq 0$ asymptotic geometry not a global solution of eoms, plus scalars blow-up

Hints at **decompactification** in UV theory \longrightarrow Embed the theory in higher dimensions!

For domain walls in [Mayer, Mohaupt](#) lift to ten dimensions gave supersymmetric ground state

Five-dimensional lift

For prepotentials of the form $F(X) = \frac{c_{ijk} X^i X^j X^k}{X^0}$ the four-dimensional theory can be embedded into five-dimensional $\mathcal{N} = 2$ supergravity

Often find that dimensional oxidation lifts global hvLif solutions to AdS solutions

[Perlmutter \(2012\)](#)
[Narayan \(2012\)](#)
[Singh \(2010\)](#)

AdS_5 has $d = 3$ and $(z, \theta) = (1, 0) \longrightarrow s \sim T^{\frac{(3-0)}{1}} = T^3$

4d gauge field lifts to 5d metric dof so want to look for stationary non-static solutions in 5d

Use 5d to 3d dimensional reduction [Dempster \(2014\)](#)

Five-dimensional gSUGRA

Lagrangian

$$e_5^{-1} \mathcal{L}_5 = -\frac{1}{2} R_{(5)} - \frac{3}{4} a_{ij}(h) \partial h^i \cdot \partial h^j - \frac{1}{4} a_{ij}(h) \mathcal{F}^i \cdot \mathcal{F}^j + \frac{1}{6\sqrt{6}} e_5^{-1} c_{ijk} \mathcal{F}^i \wedge \mathcal{F}^j \wedge \mathcal{A}^k \\ + 4 \cdot 6^{-1/3} \left[(chhh)(ch)^{-1|ij} + 3h^i h^j \right] g_i g_j$$

Reduction on S^1 gives 4d Lagrangian from earlier with “very special” prepotential

Real scalars h^i parametrise PSR manifold $\{h^i \in \mathbb{R}^n : chhh = 1\}$

Ansatz

$$ds_{(5)}^2 = \frac{1}{uv} (dx^0 + \mathcal{A}_4^0 dx^4)^2 - \frac{u}{v} (dx^4)^2 + v^2 ds_{(3)}^2 \quad \text{and} \quad \mathcal{A}^i = 0$$

Introduce $y^i = v h^i \quad \hat{g}_{ij}(y) = -\frac{3}{4v^2} a_{ij}(h) \quad \zeta^0 = -\frac{1}{\sqrt{2}} \mathcal{A}_4^0$

Dimensionally-reduced Lagrangian

Reduction over isometric x^0 and x^4 directions results in:

$$e_3^{-1} \mathcal{L}_3 = -\frac{1}{2} R_{(3)} + \hat{g}_{ij}(y) \partial y^i \cdot \partial y^j - \frac{(\partial u)^2}{4u^2} + \frac{(\partial \zeta^0)^2}{12u^2} + 6 [\hat{g}^{ij} + 4y^i y^j] g_i g_j$$

The solution

Solving 3d eoms and imposing regularity on 5d solution we find:

Line element

$$ds_{(5)}^2 = 6^{-2/3}(c\chi\chi\chi)^{-1/3}f\rho^{1/2}(dx^+ + \mathcal{A}_4^0 dx^-)^2 - 6^{1/3}(c\chi\chi\chi)^{-1/3}\frac{W\rho^{1/2}}{f}(dx^-)^2 \\ + 6^{1/3}(c\chi\chi\chi)^{2/3}\left[\frac{d\rho^2}{W\rho^2} + \rho^{1/2}(dx^2 + dy^2)\right]$$

with $f = A + \frac{\Delta}{\rho}$ $W = 1 - \frac{2B_0}{\rho}$ $\mathcal{A}_4^0 = -\sqrt{\frac{6\Delta}{\Delta + 2B_0A}}\frac{W}{f}$ $\chi^i \sim \frac{1}{g_i}$

Scalars $h^i = (c\chi\chi\chi)^{-1/3}\chi^i$ are constant (set to specific values!)

For $A \neq 0$ introduce $z = Ax^+ - \sqrt{\frac{6\Delta}{\Delta + 2B_0A}}x^-$, $t = -6^{1/2}x^-$, $r = \rho^{1/4}$

$$ds_{(5)}^2 \sim \frac{R^2}{r^2 W} dr^2 + \frac{r^2}{R^2} \left(\eta_{\mu\nu} + \frac{2B_0}{r^4} u_\mu u_\nu \right) dx^\mu dx^\nu$$

Boosted Schwarzschild
black brane

Boost
parameter

Metric is asymptotically AdS₅

$$u_t^2 = u_z^2 + 1 = \frac{\Delta}{2B_0A}$$

Fluid-gravity correspondence

Procedure for finding boundary stress-tensor from bulk data

Balasubramanian and Kraus (1999)
 Bhattacharyya, Hubeny, Minwalla, Rangamani (2007)
 Rangamani (2009)
 Hubeny (2010)

Quasi-local stress tensor

$K_{\mu\nu}$ extrinsic curvature $K = K_{\mu\nu}\gamma^{\mu\nu}$

$$\tau_{\mu\nu} = \frac{1}{8\pi G} \left[K_{\mu\nu}(\gamma) - K(\gamma)\gamma_{\mu\nu} - \frac{3}{R}\gamma_{\mu\nu} - \frac{R}{2}G_{\mu\nu}(\gamma) \right]$$

Boundary stress tensor

$$T_{\partial M}^{\mu\nu} = \lim_{r \rightarrow \infty} r^6 \tau^{\mu\nu}$$

$\gamma_{\mu\nu}$ induced metric on ∂M

We find $T_{\partial M}^{\mu\nu} = \frac{B_0 R}{8\pi G} (\eta^{\mu\nu} + 4u^\mu u^\nu)$ \longrightarrow Perfect fluid with pressure $P = \frac{B_0 R}{8\pi G}$

Conserved charge associated to KVF ξ is $P_\xi = \int_\Sigma d^3x \sqrt{\sigma} u^a T_{ab} \xi^b$

$\xi = \frac{\partial}{\partial x^-}$ gives energy density $\mathcal{E} \sim \frac{P_-}{V_2}$ $\xi = \frac{\partial}{\partial x^+}$ gives number density $N \sim \frac{P_+}{V_2}$

5d thermodynamics

Start with looking at the thermodynamics of the five-dimensional solutions

$A = 0, B_0 = 0$

Normalizable deformation giving non-trivial state in field theory

$$ds_{(5)}^2 \sim \frac{1}{z^2} (-2dx^+ dx^- + dz^2 + dx^2 + dy^2) + \Delta z^2 (dx^+)^2$$

Boundary at $z \rightarrow 0$

Scaling $(z, x, y) \mapsto \zeta(z, x, y), \quad x^+ \mapsto \zeta^{-1} x^+, \quad x^- \mapsto \zeta^3 x^- \quad z = 3$

Lifshitz scaling

Thermodynamics $T = 0, \quad s = 0, \quad \mathcal{E} = 0, \quad N \sim \Delta$

Solution obtained as 5d part of “double scaling” limit of boosted black D3 branes in IIB

Singh (2010)

$A = 0, B_0 \neq 0$

$$ds_{(5)}^2 \sim \frac{1}{z^2} \left(-2W dx^+ dx^- - \frac{B_0 W}{\Delta} (dx^-)^2 + \frac{dz^2}{W} + dx^2 + dy^2 \right) + \Delta z^2 (dx^+)^2$$

Thermodynamics $T \sim B_0^{3/4} \Delta^{-1/2}, \quad s \sim B_0^{1/4} \Delta^{1/2} \quad \mathcal{E} \sim B_0, \quad N \sim \Delta$

$A = 0$ \longrightarrow fixed number density, i.e. thermodynamic ensemble with $dN = 0$

$s(T, N) \sim T^{1/3} N^{2/3}$ Nernst behaviour, expected from Lifshitz scaling

5d thermodynamics

Now turn to the case with varying particle number

$$\underline{A \neq 0, B_0 \neq 0}$$

Thermodynamics

$$T \sim \frac{B_0^{3/4}}{\sqrt{\Delta + 2B_0A}}, \quad s \sim B_0^{1/4} \sqrt{\Delta + 2B_0A}$$

$$\mathcal{E} \sim \frac{4\Delta + 3B_0A}{A}, \quad N \sim \sqrt{\Delta(\Delta + 2B_0A)}$$

1st law?

Look at the limiting behaviour:

$$B_0 \ll \frac{\Delta}{A}$$



$$s \sim T^{1/3}$$

IR physics

$$B_0 \gg \frac{\Delta}{A}$$



$$s \sim T^3$$

UV physics

This is the UV behaviour we wanted from the 4d solution!

4d/5d relation

Make the x^+ coordinate compact $x^+ \sim x^+ + 2\pi r^+$

Dual field theory becomes effectively three-dimensional

Related to DLCQ of N=4 SYM? [Maldacena, Martelli, Tachikawa \(2008\)](#)

$$\underline{A = 0}$$

$R_{\text{phys}}^+ \sim \frac{1}{r}$ so the compactification circle shrinks in the UV

Problem with interpreting dual field theory...T-duality? [Singh \(2010\)](#)

Hyperscaling violation in 4d appears after compactification, Lifshitz scaling remains

$$\underline{A \neq 0}$$

$R_{\text{phys}}^+ \sim r \sqrt{A + \frac{\Delta}{r^4}}$ blows up in the UV \longrightarrow decompactification limit

Regulates UV behaviour of the interpolating solution in 4d!

$$Q_0^{(4d)} = \frac{1}{4} \sqrt{\frac{1}{6} \Delta (\Delta + 2B_0 A)} \propto N \quad \text{i.e. 4d charge from momentum around compact direction}$$

Conclusions and further questions

Summary

Developed a new technique for analytically finding black branes in N=2 gSUGRA


Applied it to construction of non-extremal Nernst branes

Solutions interpolate between hvLif geometries

Contribute to understanding of gauge-gravity duality for systems with hvLif behaviour

Understanding bulk side of 4d/5d relationship and regulate 4d UV behaviour

Outlook

Construct dyonic solutions and investigate resulting phase diagram  QPT?
PD, Errington, Mohaupt (in progress)

Investigate the field theory side, e.g. through transport coefficients

Applications to entanglement entropy? [Bhattacharyya, Haque, Véliz-Osorio \(2014\)](#)

Relation between 3d and 4d boundary theories, i.p. DLCQ interpretation?

Thank you!