Mass Insertion Parameters from SU(5)xS₄xU(1) model of flavour

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Introduction

The SM & SUSY Flavour Problem.
Solving it by imposing a Family symmetry.

The SU(5)xS₄xU(1) Model

The fermionic sector.

- Construction of SUSY breaking sector:
 - SCKM basis
 - Mass Insertion (MI) parameters:

Predictions for low energy MIs Vs experimental constraints.

Summary



Understanding pattern of fermion masses & mixings = = **Understanding structure of Yukawa matrices.**



Family Symmetry

Extend symmetry group with a Family symmetry G_F.

admits triplet reps
 (3 families in a triplet)

Introduce heavy scalar fields: Flavons: Φ

• couple to usual matter fields



Extend to SUSY GUTs

- Fields become <u>superfields</u>.
- Yukawa operators arise from the superpotential W:

$$W = fw\left(\frac{\Phi^n}{M^n}\right)f^cH$$

flavon vevs aligned via minimization of potential

- Kinetic terms & scalar masses arise from the Kähler potential K.
- Spartner masses & mixings must also be explained.
- Control FC processes induced by loop diags involving sfermion masses which are non-diagonal in the basis where Yukawa matrices are diagonal (SCKM basis).
- GUT models more constraining due to boundary conditions between hadronic & leptonic sectors.

$$\theta_{13}^{v} \ll \theta_{12}^{v}, \theta_{23}^{v}$$

• An interesting Family symmetry G_F would predict **TB-mixing** in the neutrino sector.

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \mathbf{0} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

<u>Neutrino mass matrix</u>:
 ✓ diagonalised by U_{TB}.
 ✓ invariant under Klein symmetry: Z^S₂ ⊗ Z^U₂

$$\theta^{v}_{13} \approx 9^{\circ}$$

$$S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2\\ 2 & -1 & 2\\ 2 & 2 & -1 \end{pmatrix} U = - \begin{pmatrix} 1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0 \end{pmatrix}$$

• Need deviations from TB.

Neutrino flavour symmetry arising from G_F

- G_F would contain the S & U generators
- preserved in the neutrino sector (m_{eff}^{v} invariant under S & U).



$SU(5) \times S_4 \times U(1)$

permutations of 4

Minimal GUT with smallest discrete group that contains S&U generators. objects

The SU(5) x S_4 x U(1) Model

$$T = \mathbf{10} = (Q, u^c, e^c)$$
 $F = \bar{\mathbf{5}} = (L, d^c)$

Field	T_3	Т	F	N	H_5	$H_{\overline{5}}$	$H_{\overline{45}}$	Φ_2^u	$\widetilde{\Phi}_2^u$	Φ_3^d	$\widetilde{\Phi}^d_3$	Φ_2^d	$\Phi^{\nu}_{3'}$	Φ_2^{ν}	Φ_1^{ν}
SU(5)	10	10	$\overline{5}$	1	5	$\overline{5}$	$\overline{45}$	1	1	1	1	1	1	1	1
S_4	1	2	3	3	1	1	1	2	2	3	3	2	3'	2	1
U(1)	0	x	y	-y	0	0	z	-2x	0	-y	-x-y-2z	z	2y	2y	2y

* <u>U(1) symmetry</u>: different flavons couple to distinct sectors at LO (according to their f label);

☆ "Leading" operators: U(1) charges add up to zero $\forall x, y, z \in Z$.

Subleading operators allowed when values of x,y,z are fixed. Forbid the unwanted ones by choosing the most appropriate values: (x,y,z)=(5,4,1)

Flavon vacuum alignment

- Introduce a set of driving fields that couple to the flavons.
- ♦ Require their F-terms to vanish: $(F^i = \partial W / \partial \phi^i = 0)$

e.g. couple the driving field X_1^d (S₄ singlet) Φ_2^d (S₄ doublet):

$$X_1^d (\Phi_2^d)^2 = 2X_1^d \Phi_{2,1}^d \Phi_{2,2}^d$$

require:
$$\frac{\partial}{\partial X_1^d} 2X_1^d \Phi_{2,1}^d \Phi_{2,2}^d = 0$$
 $\square \land \langle \Phi_2^d \rangle \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or $\langle \Phi_2^d \rangle \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Without loss of generality, pick $\Phi_{2,1}^{d} \neq 0$.

*In a similar way, all flavons are aligned through vanishing Fterms of driving fields. For the neutrino sector in particular, this process not only fixes $\langle \Phi_i^{\nu} \rangle$ but also requires that: $\varphi_1^{\nu} \sim \varphi_1^{\nu} \sim \varphi_3^{\nu}$

Flaven vacuum alignment orders of magnitude

* The Cabibbo angle requires : $\langle \Phi_2^d \rangle \sim \lambda M$, where M is a generic UV cut-off & $\lambda \sim \theta_C \sim 0.22$ is the Wolfstein parameter.

- ★ The correct size for the strange quark and the muon mass is achieved for <Φ₃^d > ~λ³ M. $(Φ^d)$ (1) $(Φ^d)$ (0) $(\tilde{Φ}^d)$ (0)
- Introducing the appropriate set of driving fields provides correlations that fix the rest:

$$\frac{\langle \Phi_2^d \rangle}{M} \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} \lambda, \quad \frac{\langle \Phi_3^d \rangle}{M} \sim \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \lambda^2, \quad \frac{\langle \tilde{\Phi}_3^d \rangle}{M} \sim \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \lambda^3$$

$$\frac{\langle \tilde{\Phi}_2^u \rangle}{M} \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} \lambda^4, \quad \frac{\langle \Phi_2^u \rangle}{M} \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} \lambda^4$$

$$\frac{\langle \Phi_2^\nu \rangle}{M} \sim \begin{pmatrix} 1 \\ 1 \end{pmatrix} \lambda^4, \quad \frac{\langle \Phi_{3'}^\nu \rangle}{M} \sim \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \lambda^4, \quad \frac{\langle \Phi_{1'}^\nu \rangle}{M} \sim \lambda^4$$

Field	X_1^d	\tilde{X}_1^d	$X^{\nu d}_{1'}$	X_1^u	Y_2^{du}	Y_2^d	Y_2^ν	$Z^{\nu}_{3'}$	V_0	V_1	V_{η}	X_1^{new}	$\tilde{X}_{\mathbf{1'}}^{new}$
SU(5)	1	1	1	1	1	1	1	1	1	1	1	1	1
S_4	1	1	1 '	1	2	2	2	3'	1	1	$1^{(\prime)}$	1	1 '
U(1)	-2	14	3	10	9	6	-16	-16	0	-8	-7	18	15

Flavon vacuum alignment CP violation & h.o. corrections

Higher order operators shift the LO vevs.

$$\begin{split} \frac{\langle \Phi_2^u \rangle}{M} &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \phi_2^u \lambda^4 + \begin{pmatrix} \delta_{21}^u \lambda^8 \\ \delta_{22}^u \lambda^5 \end{pmatrix} + \dots, \quad \frac{\langle \Phi_3^d \rangle}{M} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tilde{\phi}_2^u \lambda^4 + \begin{pmatrix} \tilde{\delta}_{21}^u \lambda^6 \\ \tilde{\delta}_{22}^u \lambda^5 \end{pmatrix} + \dots, \\ \frac{\langle \Phi_2^d \rangle}{M} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tilde{\phi}_3^d \lambda^4 + \begin{pmatrix} 0 \\ \delta_{22}^d \lambda^7 \end{pmatrix} + \dots, \quad \frac{\langle \Phi_3^d \rangle}{M} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tilde{\phi}_3^d \lambda^2 + \begin{pmatrix} \delta_{31}^d \lambda^6 \\ 0 \\ \delta_{32}^d \lambda^6 \end{pmatrix} + \dots, \\ \frac{\langle \tilde{\Phi}_3^d \rangle}{M} &= \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \tilde{\phi}_3^d \lambda^3 + \begin{pmatrix} \tilde{\delta}_{31}^d \lambda^7 \\ \tilde{\delta}_{32}^d \lambda^4 - \tilde{\delta}_{32}^d \lambda^5 \end{pmatrix} + \dots, \quad \frac{\langle \Phi_3^d \rangle}{M} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tilde{\phi}_3^u \lambda^4 + \tilde{\delta}_{33}^d \lambda^5 \end{pmatrix} + \dots, \\ \frac{\langle \Phi_2^u \rangle}{M} &= \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \phi_2^v \lambda^4 + \begin{pmatrix} \delta_{21}^v \lambda^5 \\ \delta_{22}^v \lambda^5 \end{pmatrix} + \dots, \quad \frac{\langle \Phi_{3'}^u \rangle}{M} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} \phi_{3'}^v \lambda^4 + \delta_{3'}^v \lambda^5 \end{pmatrix} \\ \frac{\langle \Phi_1^v \rangle}{M} &= \phi_1^v \lambda^4 + \delta_1^v \lambda^5 + \dots, \quad \frac{\langle \eta \rangle}{M} = \eta \lambda^4 + \delta^\eta \lambda^5 + \dots, \end{split}$$

Constructing Y^u $T = 10 = (Q, u^c, e^c)$

Write down all operators that form a singlet under all symmetries

combine up to 8 flavons with TTH_5 for the first two families & $T_3T_3H_5$ for the 3rd family.

$$\frac{1}{M^2} y_1^u TT\Phi_2^u \tilde{\Phi}_2^u H_5 + \frac{1}{M} y_2^u TT\Phi_2^u H_5 + y_t T_3 T_3 H_5 + \frac{1}{M^5} Z_1 TT(\Phi_2^d)^2 (\Phi_3^d)^3 H_5 + \frac{1}{M^5} Z_2 TT_3 (\Phi_2^d)^3 (\Phi_3^d)^2 H_5 + \frac{1}{M^3} Z_{3,4} T_3 T_3 (\Phi_3^d)^2 \Phi_{2,3'}^\nu H_5$$

Break family symmetry with non-zero flavon vevs.

$$\begin{aligned} Y^{u}_{|_{GUT}} &= \begin{pmatrix} y_{u}e^{i\theta^{y}_{u}}\lambda^{8} & 0 & 0\\ 0 & y_{c}e^{i\theta^{y}_{u}}\lambda^{4} & z_{2}^{u}e^{i\theta^{zu}_{2}}\lambda^{7}\\ 0 & z_{2}^{u}e^{i\theta^{zu}_{2}}\lambda^{7} & y_{t} \end{pmatrix} + \dots \\ y_{u}e^{i\theta^{y}_{u}} &= y_{2}^{u}\phi_{2}^{u}\tilde{\phi}_{2}^{u} + y_{1}^{u}\delta_{21}^{u}, \quad y_{c}i\theta^{y}_{u} = y_{1}^{u}\phi^{u}_{2}, \quad z_{2}^{u}e^{i\theta^{zu}_{2}} = y_{6}^{u}(\phi^{d}_{2})^{3}(\phi^{d}_{3})^{2}\\ \theta^{y}_{u} &= \theta^{y}_{c} = 2\theta^{d}_{2} + 3\theta^{d}_{3}, \quad \theta^{zu}_{2} = 3\theta^{d}_{2} + 2\theta^{d}_{3} \end{aligned}$$

Similarly, write down operators consisting of T, F & Φ^{d}_{ρ}

$$\begin{split} Y^{d}_{|GUT} &= Y^{d}_{\alpha} + Y^{d}_{\beta}, \qquad Y^{e}_{|GUT} = \left(Y^{d}_{\alpha} - 3Y^{d}_{\beta}\right)^{T} \\ Y^{d}_{\alpha} &= \begin{pmatrix} 0 & x_{2}e^{i\theta_{2}^{x}}\lambda^{5} & -x_{2}e^{i\theta_{2}^{x}}\lambda^{5} \\ -x_{2}e^{i\theta_{2}^{x}}\lambda^{5} & 0 & x_{2}e^{i\theta_{2}^{x}}\lambda^{5} \\ z^{d}_{3}e^{i\theta_{3}^{zd}}\lambda^{6} & z^{d}_{2}e^{i\theta_{2}^{zd}}\lambda^{6} & y_{b}e^{i\theta_{b}^{y}}\lambda^{2} \end{pmatrix} + \dots \\ Y^{d}_{\beta} &= \begin{pmatrix} z^{d}_{1}e^{i\theta_{1}^{zd}}\lambda^{8} & 0 & z^{d}_{11}e^{i\theta_{11}^{zd}}\lambda^{8} \\ z^{d}_{10}e^{i\theta_{10}^{zd}}\lambda^{8} & y_{s}e^{i\theta_{s}^{y}}\lambda^{4} & -y_{s}e^{i\theta_{s}^{y}}\lambda^{4} \\ 0 & z^{d}_{9}e^{i\theta_{9}^{zd}}\lambda^{7} & 0 \end{pmatrix} + \dots \end{split}$$

$$T = \mathbf{10} = (Q, u^c, e^c)$$

$$F = \mathbf{\overline{5}} = (L, d^c)$$

$$m_d : m_s : m_b \approx \lambda^4 : \lambda^2 : 1$$

$$m_e : m_\mu : m_\tau \approx (1/3)\lambda^4 : 3\lambda^2 : 1$$

$$\theta_{12}^d \approx \lambda, \quad \theta_{13}^d \approx \lambda^3, \quad \theta_{23}^d, \approx \lambda^2$$

$$\theta_{12}^e \approx (1/3)\lambda, \quad \theta_{13}^e \approx 0, \quad \theta_{23}^e, \approx 0$$

✤ Y^u almost diagonal, quark mixing coming from Y^d.

★ Georgi-Jarlskog (GJ) relations: m_b ≈m_τ, m_µ ≈3m_s, m_d ≈3m_e and GST relation: θ₁₂≈√(m_d/m_s) incorporated at LO.



Superpotential W

Gives rise to Yukawa & A-trilinear terms through $<\int d^2\theta W >$

 $\frac{\Lambda}{M_X} Hf \sum_{A \to \Phi'} a^{ff^c}_{\Phi \Phi'} \frac{\Phi \otimes \Phi'}{M^2} f^c$

picks up **F-terms** from hidden sector fields X & from flavons.

$$\langle F_{\Phi_A} \rangle_i = m_0 \, x_A \langle \Phi_A \rangle_i$$

trilinears have same structure as Yukawas but different O(1) coefs.

trilinears & Yukawas can not be simultaneously diagonalised.



Origin of **off-diagonalities** in the SCKM basis

Kähler potentials
$$K_F, K_T, K_N$$
4 θ K> give rise to kinetic terms & soft scalar masses $\mathcal{L}_K \supset \tilde{K}_{ij} \left(\partial_\mu \varphi_i^* \partial^\mu \varphi_j + i \eta_i^* \partial_\mu \bar{\sigma}^\mu \eta_j \right)$ Kähler metric: $\tilde{K}_{ij} = \frac{\partial^2 K}{\partial f_i^\dagger f_j} \Big|_{f=\varphi}$ generic sfields

Flavon expansion :

$$K_F = F^{\dagger} \left[\left(c_0^{K_F} + c_0^{M_F} \frac{X^{\dagger} X}{M_X^2} \right) \mathbb{1}_3 + \sum_{\Phi \Phi'} \left(c_{\Phi \Phi'}^{K_F} + c_{\Phi \Phi'}^{M_F} \frac{X^{\dagger} X}{M_X^2} \right) \frac{\Phi \otimes \Phi'}{M^2} \right] F$$

✤Kähler metrics & soft masses: same structure, different O(1) coefs.

Generation of off-diagonalities is inevitable.

 Work in a basis where:
$$\mathbf{\tilde{K}^{ij}=1}$$
. **Canonical Normalisation**

Canonical Normalisation: change of basis such that: $(P^{\dagger})^{-1} \overrightarrow{K} P^{-1} = 1$

- Bring all quantities into that basis.
- *Y^u_c: zero entries are populated; (23) & (32) entries reduced by two orders of λ .
- * Y_c^v : (12), (21) & (33) entries also reduced by two orders of λ.
- \clubsuit Rest of the effects just consist of changing the O(1) coefs.

Successful fermionic masses & mixings survive.

Now the off-diagonalities in the soft sector have to be controlled in order to lead to predictions that agree with the FCNC bounds.

The SUSY Flavour Problem

The SUSY Flavour Problem

generation mixing...

FC

u, c, t

S



Only through loops with charged particles

The SUSY Flavour Problem

Mass Insertion approximation

- Work in Super-CKM basis (diagonal m_d)
- * gluino vertex diagonal in flavour but non-diagonal \tilde{m}_{d}^{2} .

Approximate squark propagator.

MI

$$\left(\tilde{m}^d\right)_{ij}^2 = (\tilde{m}^d)^2 \,\mathbbm{1} + \Delta_{ij}^d$$

$$\frac{i}{\left(\left(k^2 - (\tilde{m}^d)^2\right) 1\!\!1 - \Delta^d\right)_{ij}} \approx i \frac{\delta_{ij}}{k^2 - (\tilde{m}^d)^2} + i \frac{\Delta^d_{ij}}{(k^2 - (\tilde{m}^d)^2)^2} + \dots$$





The SUSY Flavour Problem

MI

Mass Insertion approximation

$$(\delta^{d}_{AB})_{ij} = \frac{(\Delta^{d}_{AB})_{ij}}{(\tilde{m}^{d})^{2}}, \quad \{A, B\} = \{L, R\}$$
$$\mathcal{H}^{\Delta F=2}_{\text{eff}} = \sum_{i=1}^{5} C_{i} Q_{i} + \sum_{i=1}^{3} \tilde{C}_{i} \tilde{Q}_{i} + \text{h.c.},$$

$$\begin{split} C_{1}^{\tilde{g}} &\simeq -\frac{\alpha_{s}^{2}}{\tilde{m}^{2}} \big[\left(\delta_{d}^{LL} \right)_{21} \big]^{2} g_{1}^{(1)}(x_{g}), \\ \tilde{C}_{1}^{\tilde{g}} &\simeq -\frac{\alpha_{s}^{2}}{\tilde{m}^{2}} \big[\left(\delta_{d}^{RR} \right)_{21} \big]^{2} g_{1}^{(1)}(x_{g}), \\ C_{4}^{\tilde{g}} &\simeq -\frac{\alpha_{s}^{2}}{\tilde{m}^{2}} \big[\left(\delta_{d}^{LL} \right)_{21} \left(\delta_{d}^{RR} \right)_{21} \big] g_{4}^{(1)}(x_{g}), \\ C_{5}^{\tilde{g}} &\simeq -\frac{\alpha_{s}^{2}}{\tilde{m}^{2}} \big[\left(\delta_{d}^{LL} \right)_{21} \left(\delta_{d}^{RR} \right)_{21} \big] g_{5}^{(1)}(x_{g}), \end{split}$$

 $x_g = M_{\tilde{\varrho}}^2 / \tilde{m}^2$

- Since the observed FCNCs are strongly suppressed, experiment sets strong bounds on these parameters.
- ★ In our particular example, the relevant observable is: $M_{12}^{(K^0)} \equiv \langle K^0 | \mathcal{H}_{\text{eff}}^{\Delta F=2} | \bar{K}^0 \rangle$

Need to check whether our model predicts MIs that agree with the current bounds!

SCKM basis Change to the basis where Yukawas are diagonal: e.g. $\tilde{Y}^{d}_{|_{GUT}} = (U^{d}_{L})^{\dagger} Y^{d_{C}}_{|_{GUT}} U^{d}_{R} = \begin{pmatrix} \frac{x_{2}}{y_{s}} \lambda^{6} & 0 & 0\\ 0 & y_{s} \lambda^{4} & 0\\ 0 & 0 & y_{b} \lambda^{2} \end{pmatrix} + \dots$ $\frac{\tilde{A}^d_{|_{GUT}}}{A_0} = (U^d_L)^{\dagger} \frac{A^d_{C_{|_{GUT}}}}{A_0} U^d_R = \begin{pmatrix} \tilde{a}^d_{11}\lambda^6 & \tilde{a}^d_{12}\lambda^5 & \tilde{a}^d_{13}\lambda^5 \\ \tilde{a}^d_{21}\lambda^5 & \tilde{a}^d_{22}\lambda^4 & \tilde{a}^d_{23}\lambda^4 \\ \tilde{a}^d_{21}\lambda^6 & \tilde{a}^d_{22}\lambda^6 & \tilde{a}^d_{22}\lambda^2 \end{pmatrix} + \dots$ $\tilde{a}_{11}^d = \frac{\tilde{x}_2^2}{u_s} \left(2 \frac{\tilde{x}_2^a}{\tilde{x}_2} e^{i(\theta_2^{\tilde{x}_a} - \theta_2^{\tilde{x}_2})} - \frac{a_s}{u_s} e^{i(\theta_s^a - \theta_s^y)} \right), \quad \tilde{a}_{22}^d = a_s e^{i(\theta_s^a - \theta_s^y)}, \quad \tilde{a}_{33}^d = a_b e^{i(\theta_b^a - \theta_b^y)},$ $\tilde{a}_{12}^{d} = \tilde{x}_{2} e^{i(\theta_{2}^{\tilde{x}} - \theta_{s}^{y})} \left(\frac{\tilde{x}_{2}^{a}}{\tilde{x}_{2}} e^{i(\theta_{2}^{\tilde{x}_{a}} - \theta_{2}^{\tilde{x}})} - \frac{a_{s}}{u_{s}} e^{i(\theta_{s}^{a} - \theta_{s}^{y})} \right), \dots$

If the trilinears were aligned with the Yukawas, their off-diag terms would drop out, while the diag ones would converge to the associated Yukawa eigenvalues, up to a global factor.



Similarly, if the coefs of M_F^2 were universally proportional to the associated K_F ones, then canonical normalisation would render the mass matrix diagonal. This would not happen to M_T^2 however due to the splitting of the first two and the third generations ($b_{01} \neq b_{02}$).

such a tuning can not be justified focus on producing small off-diagonalities, to stay in agreement with FCNC bounds.

Define 3x3 full sfermion matrices as:

$$\begin{split} m_{\tilde{f}_{LL}}^2 &= (\tilde{m}_{f}^2)_{LL} + \tilde{Y}_{f} \tilde{Y}_{f}^{\dagger} \upsilon_{u,d}^2 \quad , \quad m_{\tilde{f}_{RR}}^2 = (\tilde{m}_{f}^2)_{RR} + \tilde{Y}_{f}^{\dagger} \tilde{Y}_{f} \upsilon_{u,d}^2 \\ m_{\tilde{f}_{LR}}^2 &= \tilde{A}_{f} \upsilon_{u,d} - \mu \tilde{Y}_{f} \upsilon_{d,u} \quad , \quad m_{\tilde{f}_{RL}}^2 = \tilde{A}_{f}^{\dagger} \upsilon_{u,d} - \mu \tilde{Y}_{f}^{\dagger} \upsilon_{d,u} \end{split}$$

Theoretical predictions in terms of the dim/less parameters:

$$(\delta_{LL}^{f})_{ij} = \frac{(m_{\tilde{f}_{LL}}^2)_{ij}}{\langle m_{\tilde{f}} \rangle_{LL}^2}, \ (\delta_{RR}^f)_{ij} = \frac{(m_{\tilde{f}_{RR}}^2)_{ij}}{\langle m_{\tilde{f}} \rangle_{RR}^2}, (\delta_{LR}^f)_{ij} = \frac{(m_{\tilde{f}_{LR}}^2)_{ij}}{\langle m_{\tilde{f}} \rangle_{LR}^2}, \ (\delta_{RL}^f)_{ij} = \frac{(m_{\tilde{f}_{RL}}^2)_{ij}}{\langle m_{\tilde{f}} \rangle_{RL}^2},$$

$$\sqrt{(m_{\tilde{f}}\rangle_{AB}^2)} = \sqrt{(m_{\tilde{f}_{AA}}^2)_{ii}(m_{\tilde{f}_{BB}}^2)_{jj}}$$

$$(\delta^{u})_{LL} = \begin{pmatrix} 1 \ \lambda^{4} \ \lambda^{6} \\ \cdot \ 1 \ \lambda^{5} \\ \cdot \ \cdot \ 1 \end{pmatrix}, \quad (\delta^{u})_{RR} = \begin{pmatrix} 1 \ \lambda^{4} \ \lambda^{6} \\ \cdot \ 1 \ \lambda^{5} \\ \cdot \ \cdot \ 1 \end{pmatrix}, \quad (\delta^{u})_{LR} = \begin{pmatrix} \lambda^{8} \ 0 \ 0 \\ 0 \ \lambda^{4} \ \lambda^{7} \\ 0 \ \lambda^{7} \ 1 \end{pmatrix}$$

$$(\delta^d)_{LL} = \begin{pmatrix} 1 \ \lambda^3 \ \lambda^4 \\ \cdot \ 1 \ \lambda^2 \\ \cdot \ \cdot \ 1 \end{pmatrix}, \quad (\delta^d)_{RR} = \begin{pmatrix} 1 \ \lambda^4 \ \lambda^4 \\ \cdot \ 1 \ \lambda^4 \\ \cdot \ \cdot \ 1 \end{pmatrix}, \quad (\delta^d)_{LR} = \begin{pmatrix} \lambda^6 \ \lambda^5 \ \lambda^5 \end{pmatrix} \\ \lambda^5 \ \lambda^4 \ \lambda^4 \\ \lambda^6 \ \lambda^6 \ \lambda^2 \end{pmatrix}$$

$$(\delta^e)_{LL} = \begin{pmatrix} 1 \ \lambda^4 \ \lambda^4 \\ \cdot \ 1 \ \lambda^4 \\ \cdot \ \cdot \ 1 \end{pmatrix}, \quad (\delta^e)_{RR} = \begin{pmatrix} 1 \ \lambda^3 \ \lambda^4 \\ \cdot \ 1 \ \lambda^2 \\ \cdot \ \cdot \ 1 \end{pmatrix}, \quad (\delta^e)_{LR} = \begin{pmatrix} \lambda^6 \ \lambda^5 \ \lambda^6 \\ \lambda^5 \ \lambda^4 \ \lambda^6 \\ \lambda^5 \ \lambda^4 \ \lambda^2 \end{pmatrix}$$

Small off-diagonalities, close to MFV but...small enough?
 RG run down to the low energy scale where experiments are performed and compare with given bounds.

Effects of RG running

LLog approx: $M_{GUT} \approx 2 \times 10^{16} \,\text{GeV}, \ M_R \approx 10^{14} \,\text{GeV}, \ M_{Low} \approx 10^3 \,\text{GeV}$

$$\tilde{Y}^{d}_{|_{Low}} \approx \begin{pmatrix} 1 + R^{y}_{d} & 0 & 0\\ 0 & 1 + R^{y}_{d} & 0\\ 0 & 0 & 1 + R^{y}_{b} \end{pmatrix} \tilde{Y}^{d}_{|_{GUT}} + \begin{pmatrix} 0 & 0 & \tilde{y}^{d}_{13}\lambda^{6}\\ 0 & 0 & \tilde{y}^{d}_{23}\lambda^{4}\\ 0 & \tilde{y}^{d}_{32}\lambda^{6} & 0 \end{pmatrix}$$

$$\tilde{y}_{13}^{d} = -\eta \, y_t^2 y_b U_{L_{31}}^{d*} \quad , \quad \tilde{y}_{23}^{d} = \eta \, y_t^2 y_b U_{L_{23}}^{d} \quad , \quad \tilde{y}_{32}^{d} = \eta \, y_t^2 y_s U_{L_{23}}^{d*}$$
$$R_d^y = \eta \frac{44}{5} g_U^2, \quad R_b^y = R_d^y - \eta \, y_t^2$$

SCKM transformation before running \longrightarrow generation of off-diag elements in Yukawas, proportional to quark masses & V_{CKM} elements. Still small, can be treated as perturbation.



$$\begin{split} & \frac{(\tilde{m}_{d}^{2})_{LL_{|_{Low}}}}{m_{0}^{2}} \approx \frac{(\tilde{m}_{d}^{2})_{LL_{|_{GUT}}}}{m_{0}^{2}} + (6.5\frac{M_{1/2}^{2}}{m_{0}^{2}} + T_{L}^{d}) 1 - 2\eta R_{q} \begin{pmatrix} 0 & 0 & U_{L_{31}}^{d*} \lambda^{4} \\ \cdot & 0 & - U_{L_{23}}^{d} \lambda^{2} \\ \cdot & \cdot & 2 \end{pmatrix} \\ & \frac{(\tilde{m}_{d}^{2})_{RR_{|_{Low}}}}{m_{0}^{2}} \approx \frac{(\tilde{m}_{d}^{2})_{RR_{|_{GUT}}}}{m_{0}^{2}} + (6.1\frac{M_{1/2}^{2}}{m_{0}^{2}} + T_{R}^{d}) 1 \\ & R_{q} = (2b_{02} + c_{H_{u}}) y_{t}^{2} + \frac{A_{0}^{2}}{m_{0}^{2}} a_{t}^{2}, \\ T_{L}^{d} = \frac{1}{20}T + \left(-\frac{1}{2} + \frac{1}{3}\sin^{2}(\theta_{W})\right) \cos(2\beta) M_{Z}^{2}, \\ T_{R}^{d} = \frac{1}{5}T - \frac{1}{3}\sin^{2}(\theta_{W})\cos(2\beta) M_{Z}^{2}, \quad T = \frac{1}{4\pi^{2}} \int_{\ln(M_{GUT})} g_{U}^{2}(c_{H_{u}} - c_{H_{d}}) m_{0}^{2} \end{split}$$

high scale off-diagonalities not significantly affected but diagonal elements increased

Effects of RG running

Low energy Mis suppressed as sfermion masses get larger with running. *again work in the basis with diagonal Yukawas*

$$\begin{split} & \left(\delta_{ij}^{d}\right)_{RR} = \frac{1}{\left(p_{R}^{d}\right)^{2}} \left(\delta_{ij}^{d}\right)_{RR_{\mid GUT}}, \qquad \left(\delta_{12}^{d}\right)_{LL} = \frac{b_{01}}{\left(p_{L^{1G}}^{d}\right)^{2}} \left(\delta_{12}^{d}\right)_{LL_{\mid GUT}}, \\ & \left(\delta_{13(23)}^{d}\right)_{LL} = \frac{\sqrt{b_{01}b_{02}}}{\left(p_{L^{1G}}^{d}p_{L^{3G}}^{d}\right)} \left(1 + \eta \left(y_{t}^{2} - \frac{2R_{q}}{b_{02} - b_{01}}\right)\right) \left(\delta_{13(23)}^{d}\right)_{LL_{\mid GUT}}, \\ & \left(\delta_{13(23)}^{d}\right)_{LL} = \sqrt{1 + 6.1} \frac{M_{1/2}^{2}}{m_{0}^{2}}, \qquad p_{L^{1G}}^{d} = \sqrt{b_{01} + 6.5} \frac{M_{1/2}^{2}}{m_{0}^{2}}, \\ & p_{R}^{d} = \sqrt{1 + 6.1} \frac{M_{1/2}^{2}}{m_{0}^{2}}, \qquad p_{L^{1G}}^{d} = \sqrt{b_{01} + 6.5} \frac{M_{1/2}^{2}}{m_{0}^{2}}, \\ & p_{L^{3G}}^{d} = \sqrt{b_{02} + 6.5} \frac{M_{1/2}^{2}}{m_{0}^{2}} - 2\eta R_{q}, \\ & p_{L^{3G}}^{d} = \sqrt{b_{02} + 6.5} \frac{M_{1/2}^{2}}{m_{0}^{2}} - 2\eta R_{q}, \\ & F_{L}^{\nu} = \frac{1}{2} \left(\frac{1}{2} \tilde{Y}_{\nu} \tilde{Y}_{\nu}^{\dagger} (\tilde{m}_{e}^{2})_{LL} + \frac{1}{2} (\tilde{m}_{e}^{2})_{LL} \tilde{Y}_{\nu} \tilde{Y}_{\nu}^{\dagger} + \tilde{Y}_{\nu} (\tilde{m}_{N}^{2})_{RR} \tilde{Y}_{\nu}^{\dagger} + (m_{H_{u}}^{2}) \tilde{Y}_{\nu} \tilde{Y}_{\nu}^{\dagger} + \tilde{A}_{\nu} \tilde{A}_{\nu}^{\dagger}\right) \end{split}$$

Numerical estimates

- SM fit for fermionic sector and scan over $t_{\beta} \in [5, 25]$, $M_{1/2} \in [300, 3000]$, $m_0 \in [50, 10000]$, $A_0 \in [-3,3]$ m_0 & unknown SUSY coefficients in $\pm [0.5, 2]$.
- μ parameter fixed through:



radiative corrections





Numerical estimates

Parameter

- $|(\delta^{e}_{LL})_{12}|$
- $|(\delta^{e}_{LR})_{12}|$
- $|(\delta^{e}_{LL})_{13,23}|$
- $|(\delta^{e}_{LR})_{13,31,23}|$
- $|(\delta^{e}_{LR})_{32}|$
- $|(\delta^{e}_{RR})_{12}|$
- $|(\delta^{e}_{RR})_{13}|$
- $|(\delta^{e}_{RR})_{23}|$

Prediction $O(10^{-6}, 10^{-2})$ $O(10^{-9}, 10^{-4})$ $O(10^{-6}, 10^{-2})$ $O(10^{-9}, 10^{-4})$ $O(10^{-8}, 10^{-3})$ $O(10^{-5}, 10^{-2})$ $O(10^{-5}, 5*10^{-4})$ $O(10^{-3}, 10^{-1})$

Bound $O(10^{-5}, 10^{-4})$ $O(10^{-6}, 10^{-5})$ $O(10^{-3}, 10^{-2})$ $O(10^{-2}, 10^{-1})$ $O(10^{-2}, 10^{-1})$ $O(10^{-3}, 10^{-2})$ $O(10^{-1}, 1)$ $O(10^{-1}, 1)$

Numerical estimates

Parameter

Prediction

- $|(\delta^{d}_{LL})_{23}|$
- $|(\delta^{d}_{LL})_{12}|$
- $|\text{Im}(\delta^{d}_{LR})_{12}|$
- $|(\delta^{d}_{RR})_{23}|$
- $|(\delta^{d}_{RL})_{23}|$
- $|(\delta^{d}_{LR})_{23}|$
- $|(\delta^{u}_{LR})_{23}|$

 $O(10^{-8}, 5*10^{-2})$ $O(10^{-5}, 5*10^{-2})$ $O(10^{-7}, 10^{-6})$ $O(10^{-4}, 10^{-3})$ $O(10^{-7}, 10^{-6})$ $O(10^{-6}, 10^{-5})$ $O(10^{-6})$

 $O(10^{-2}, 10^{-1})$ $O(10^{-3}, 10^{-2})$ $O(10^{-4}, 10^{-3})$ $O(10^{-1}, 1)$ $O(10^{-2})$ $O(10^{-3}, 10^{-2})$ 0.3 (MS~1TeV) 0.1 (MS~3TeV)

Bound

Phenomenological Implications

Bounds on MIs available in the literature.

They are placed by demanding that the contribution of each MI to an observable does not exceed the relevant experimental limit.

Bounds taken from: arXiv: 1405.6960, arXiv: 1304.2783, arXiv: 1207.3016

★ Comparison with our predictions suggests study of phenomenology related $\mu \rightarrow e\gamma$, edms and b→s transitions.

Phenome $\mu \rightarrow e\gamma$ plications

* Strongest constraint from $Br(\mu \rightarrow e\gamma)$

In the SM suppressed by small neutrino masses

$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_{i} U_{\mu i}^{*} U_{e i} \frac{m_{\nu_{i}}^{2}}{m_{W}^{2}} \right|^{2} \lesssim 10^{-54}$$

* The SUSY contribution through bino, bino-higgsino & wino-higgsino loops, involves the δ^{e}_{12} parameters.



Phenome $\mu \rightarrow e\gamma$ plications

$$BR(\mu \to e \ \gamma) = 3.4 \times 10^{-4} \times \frac{1}{4} \left[\hat{x} \frac{\mu^2 t_{\beta}^2}{m_0^6} M_W^4 \times \left(\left| (\delta_{LL}^e)_{12} \left(-(\delta_{LR}^e)_{22} \frac{\tilde{m}_{LL}^e \tilde{m}_{RR}^e}{\mu t_{\beta} m_{\mu}} A'_{B,L} + \frac{1}{2} A'_L + A'_2 \right) + (\delta_{LR}^e)_{12} \frac{\tilde{m}_{LL}^e \tilde{m}_{RR}^e}{\mu t_{\beta} m_{\mu}} A_B \right|^2 + \left| (\delta_{RR}^e)_{12} \left(-(\delta_{LR}^e)_{22}^* \frac{\tilde{m}_{LL}^e \tilde{m}_{RR}^e}{\mu t_{\beta} m_{\mu}} A'_{B,R} - A'_R \right) + (\delta_{LR}^e)_{21}^* \frac{\tilde{m}_{LL}^e \tilde{m}_{RR}^e}{\mu t_{\beta} m_{\mu}} A_B \right|^2 \right) \right|^{14} \left[\frac{\log(5.7 \ 10^{-13})/\log(\mathrm{BR}(\mu \to e \ \gamma)))}{14} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{14} + \frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{14} + \frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma))} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)} \right]^{14} \left[\frac{\log(5.7 \ 10^{-13})}{\log(\mathrm{BR}(\mu \to e \ \gamma)} \right]^$$



 $x=(M_{1/2}/m_0)^2$, A_i: dim/less loop functions

 $(\delta^{e}_{12})_{LL}$, $(\delta^{e}_{12})_{LR}$: dominant contributions

Pheno electron edm cations $\frac{d_e}{e} = \frac{\alpha M_1}{8\pi m_0^4 \cos^2 \theta_W} \tilde{m}_{LL}^e \tilde{m}_{RR}^e$ $\operatorname{Im} \left[-\frac{\delta_{LR}^e}{(\delta_{LR}^e)_{11}A_B} + (\delta_{LL}^e)_{1i}(\delta_{LR}^e)_{i1}A'_{B,L} + \frac{\delta_{LR}^e}{(\delta_{RR}^e)_{1i}(\delta_{RR}^e)_{i1}A'_{B,R}} - \frac{\delta_{LL}^e}{((\delta_{LL}^e)_{1i}(\delta_{RR}^e)_{i1} + (\delta_{LR}^e)_{1i}(\delta_{RL}^e)_{ij}(\delta_{LR}^e)_{j1}} + (\delta_{LL}^e)_{1i}(\delta_{LR}^e)_{ij}(\delta_{RR}^e)_{j1} + (\delta_{LR}^e)_{1i}(\delta_{RR}^e)_{ij}(\delta_{RR}^e)_{j1}} + (\delta_{LL}^e)_{1i}(\delta_{LR}^e)_{ij}(\delta_{RR}^e)_{j1} + (\delta_{LR}^e)_{1i}(\delta_{RR}^e)_{ij}(\delta_{RR}^e)_{j1}} \right] A''_B \right]$



* If the phases of the trilinear sector are the same as the corresponding Yukawa ones, $(\delta^{e}_{LR})_{11} \sim \lambda^{6}$ dominates (green points)

*Alternatively, $(\delta^{e}_{LR})_{12} (\delta^{e}_{RR})_{21} \sim \lambda^{9}$ dominates (blue points).



Phenome B-mixing plications



$$h_q = \left| \frac{M_{12}^{q,SUSY}}{M_{12}^{q,SM}} \right|, \quad 2\sigma_q = \operatorname{Arg}[M_{12}^{q,SUSY}] + \phi_q^{SM}$$

Within current & future experimental limits. Similar results for σ_d -h_d.



Phenome R_mixing highlightions Time-dependent CP Asymmetry

$$A_{CP}^{q}(f,t) \equiv \frac{\Gamma\left(\bar{B}_{q}(t) \to f\right) - \Gamma\left(B_{q}(t) \to f\right)}{\Gamma\left(\bar{B}_{q}(t) \to f\right) + \Gamma\left(B_{q}(t) \to f\right)} \approx S_{f} \sin(2|M_{12}^{q}|t)$$

$$S_f = \sin\left(\phi_{SM}^q + Arg[1 + h_q e^{2i\sigma_q}]\right)$$







- SU(5) x S₄xU(1) Flavour model successfully predicts the fermionic masses and mixing angles.
- Considering canonical normalisation effects does not spoil the original features of the fermionic sector.
- Predicted off-diagonalities of soft terms (and MIs) small at the GUT scale.
- * Strongest constraint from $\mu \rightarrow e\gamma$. $(\delta^{e}_{12})_{AB}$ around their upper limits.
- ♦ Comparison with the rest of the bounds points towards phenomenological study of edms, ϵ_{κ} and b→s transitions.

Thank you for your attention