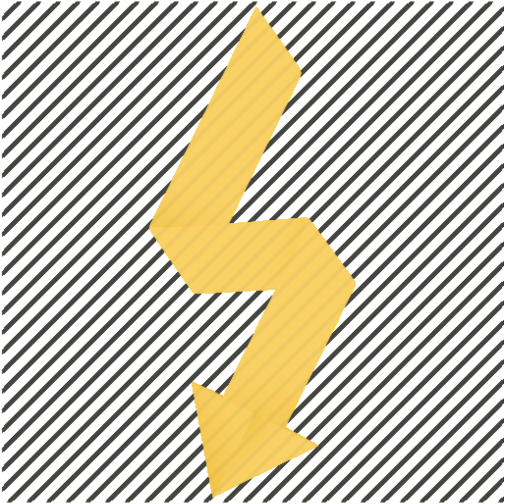
The Contradiction



Semi-Realistic Heterotic-String Vacua

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String Theory Seminar May 2015

"Patience is bitter, but its fruit is sweet."
Jean-Jacques Rousseau

Content

- Motivation
- Free Fermionic Construction
- Non-Supersymmetric Tachyon Free Model
- Results
- Observations & Problems

Motivation

- To incorporate GRAVITY with the Standard Model gauge group $SU(3) \times SU(2) \times U(1)$.

Always the 16 of $SO(10)$

	$SU(3)$	$SU(2)$
Q	$\square$	$\square$
u	$\overline{\square}$	1
d	$\overline{\square}$	1
L	1	$\square$
e	1	1
ν	1	1

Why (Bosonic) String Theory Is Not The Whole Story?

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- The ground state of the spectrum always contains a tachyon. As a consequence, the vacuum is unstable.

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Two major setbacks

- The ground state of the spectrum always contains a tachyon. As a consequence, the vacuum is unstable.
- Does not contain space-time fermions.

Why Superstrings?

Supersymmetry is the symmetry that interchanges bosons and fermions.

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Heterotic strings are hybrid strings with either

- left-moving sector being supersymmetric and right-moving sector being bosonic or
- left-moving sector being bosonic and right-moving sector being supersymmetric.
- There are two heterotic string theories, one associated to the gauge group

$$E_8 \times E_8$$

and the other to

$$SO(32)$$

or more precisely

$$Spin(32)/\mathbb{Z}_2$$

which is the double cover of $SO(32)$.

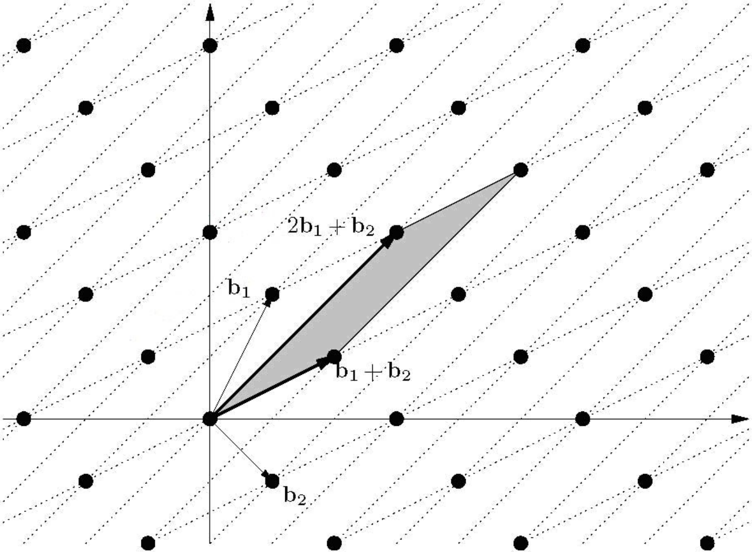
Lattice

A lattice is defined as a set of points in a vector space $\mathcal{V}$ lets say $\mathbb{R}^{(p,q)}$ with Lorentzian inner product of the form

$$\Lambda = \left\{ \sum_{i=1}^{m} n_i e_i, n_i \in \mathbb{Z} \right\}$$

where $m = p + q$ and $\{e_i\}$ are the basis vectors which form the canonical basis of Λ. The metric on this lattice which contains information about the lengths and angles between the basis vectors is defined as

$$g_{ij} = e_i \cdot e_j.$$


$2b_1 + b_2$
b_1
$b_1 + b_2$
b_2

The Dual Lattice

The dual lattice is given by

$$\Lambda^* = \left\{ \sum_{i=1}^{m} n_i e_i^*, n_i \in \mathbb{Z} \right\}$$

where $\{e_i^*\}$ is the canonical basis of the dual lattice.
The basis vectors of the dual lattice are chosen in a such a way as to satisfy the condition

$$e_i^* \cdot e_j = \delta_{ij}.$$

he metric of the dual lattice is given by

$$g_{ij}^* = e_i^* \cdot e_j^*$$

which is simply the inverse of g_{ij}.

The Different Kinds

- A lattice is called *integral* if $v \cdot \omega \in \mathbb{Z} \; \forall \; v, \; \omega \in \Lambda$.

- A lattice is called *even* if Λ is integral and v^2 is even for all $v \in \Lambda$.

The Different Kinds

- A lattice is called *self-dual* if $\Lambda = \Lambda^*$.

The Different Kinds

- In 16 dimensions, there are only two even self dual lattices.

The Basic Aim

The Goal of the Free Fermionic Construction

- 4 Flat Space-Time Dimensions
- $N = 1$ SUSY
- 3 Chiral Generations

The Free Fermionic Construction

Properties

- Conformally Invariant
- Decoupling of Left & Right Moving Modes
- D = 4 Theory

Result

- $C_L = -26 + 11 + D + \frac{D}{2} + \frac{N_{f_L}}{2} = 0$

  $\Longrightarrow$ 18 left-moving real fermions
- $C_R = -26 + D + \frac{N_{f_R}}{2} = 0$

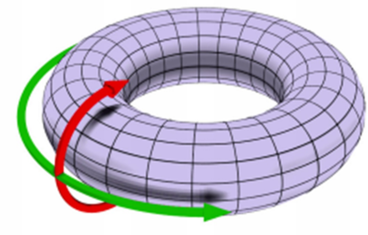
  $\Longrightarrow$ 44 right-moving real fermions

The Free Fermionic Construction

- Partition function is used to include all physical states

$$Z = \sum_{\alpha,\beta} C \begin{pmatrix} \alpha \\ \beta \end{pmatrix} Z\left[\alpha, \beta\right]$$

- Taking the one-loop partition function transforms the worldsheet into a torus.



It is around the two non-contractible loops of this torus that the fermions on being parallel transported will pick up a phase.

The Free Fermionic Construction

$$\alpha = \left\{ \psi_\mu^{1,2}, \chi^i, y^i, \omega^i | \overline{y}^i, \overline{\omega}^i, \overline{\psi}^{1,..,5}, \overline{\eta}^{1,2,3}, \overline{\phi}^{1,..,8} \right\}$$

where $i = 1, ..., 6$

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where $i = 1, ..., 6$

- Left-movers
  - X_L^μ, $\mu = 1, 2$ 2 transverse coordinates
  - ψ_L^μ, $\mu = 1, 2$ The fermionic partners
  - Ω^j, $j = 1, .., 18$ 18 internal real fermions

The Free Fermionic Construction

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- Left-movers
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  - ψ_L^μ, $\mu = 1, 2$ The fermionic partners
  - Ω^j, $j = 1, .., 18$ 18 internal real fermions
- Right-movers
  - X_R^μ, $\mu = 1, 2$ 2 transverse coordinates
  - $\overline{\Omega}^j$, $j = 1, .., 44$ 44 internal real fermions

$SU(2)^6$ - The 18 Dimensional Semi-Simple Lie Algebra

SUSY is non-linearly realized. The supercharge is

$$T_F = \psi^\mu \partial X_\mu + f_{IJK} \chi^I \chi^J \chi^K = \psi^\mu \partial X_\mu + \sum_I \chi^I y^I \omega^I$$

where f_{IJK} are the structure constants of a semi-simple Lie group G with 18 generators.

The Space-Time Spin Statistics Index

$$\delta_\alpha = \begin{cases} 1 \Leftrightarrow \alpha(\psi^\mu_{1,2}) = 0 \\ -1 \Leftrightarrow \alpha(\psi^\mu_{1,2}) = 1 \end{cases}$$

The Free Fermionic Construction

- The ABK Rules
  - $\sum_i m_i b_i = 0$
  - $N_{ij} \cdot b_i \cdot b_j =$0 mod 4
  - $N_i \cdot b_i \cdot b_i =$0 mod 8
  - $1 \in \Xi$, (Ξ is the Abelian additive group)
  - Even number of fermions
- One-Loop Phases
  - $C\binom{b_i}{b_j} = \pm 1$ or $\pm i$
- GSO Projection
  - $e^{i\pi b_i \cdot F_\alpha} |s\rangle_\alpha = \delta_\alpha C\binom{\alpha}{b_i}^* |s\rangle_\alpha$
- Virasoro Level-Matching Condition
  - $M_L^2 = -\frac{1}{2} + \frac{\alpha_L^2}{8} + \sum v_L = -1 + \frac{\alpha_R^2}{8} + \sum v_R = M_R^2$

The Frequency of Fermions

The fermions transform under the parallel transport as

$$f \to -e^{i\pi\alpha(f)} f$$

their frequency being given by

$$\nu_f = \frac{1+\alpha(f)}{2}.$$

Due to the periodicity of the phases we write the frequency more precisely as

$$\nu_f = \frac{1+\alpha(f)}{2} + F$$

Then the $U(1)$ charge is given by

$$Q_\nu(f) = \zeta(0, 1-\nu) = -B_1(1-\nu) = \nu - \frac{1}{2} = \frac{1}{2}\alpha(f) + F, \quad B_1 = +\frac{1}{2}$$

An Example: The NAHE Set

The NAHE set is the set of basis vectors

$$B = \{\mathbb{1}, \mathbf{S}, \mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}\}$$

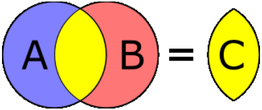
where

$$\begin{aligned}
\mathbb{1} &= \{\psi_\mu^{1,2}, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\dots,8}\}, \\
\mathbf{S} &= \{\psi_\mu^{1,2}, \chi^{1,\dots,6}\}, \\
\mathbf{b_1} &= \{\psi_\mu^{1,2}, \chi^{1,2}, y^{3,\dots,6} | \bar{y}^{3,\dots,6}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{1}\}, \\
\mathbf{b_2} &= \{\psi_\mu^{1,2}, \chi^{3,4}, y^{1,2}, \omega^{5,6} | \bar{y}^{1,2}, \bar{\omega}^{5,6}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{2}\}, \\
\mathbf{b_3} &= \{\psi_\mu^{1,2}, \chi^{5,6}, \omega^{1,\dots,4} | \bar{\omega}^{1,\dots,4}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{3}\}.
\end{aligned}$$

The NAHE: The Space-Time Spin Statistics Index

$$\begin{aligned}\delta_{\mathbb{1}} &= \delta_{\mathbf{S}} \\ &= \delta_{\mathbf{b_1}} \\ &= \delta_{\mathbf{b_2}} \\ &= \delta_{\mathbf{b_3}} \\ &= -1\end{aligned}$$

$\alpha \cdot \beta = (\#L - \#R) n_{\mathbb{C}}(\alpha \cap \beta)$
$\#L > \#R = +$, $\#L < \#R = -$, $\#L \neq \#R$



For example, consider

$$1 + b_1 + b_2 + b_3$$

the union is

$$\mathbb{1} = \{\psi_\mu^{1,2}, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\dots,8}\}$$

Now subtract, b_1, b_2 and b_3

$$\mathbb{1} = \{\cancel{\psi}_\mu^{1,2}, \cancel{\chi}^{1,\dots,6}, \cancel{y}^{1,\dots,6}, \cancel{\omega}^{1,\dots,6} | \cancel{\bar{y}}^{1,\dots,6}, \cancel{\bar{\omega}}^{1,\dots,6}, \cancel{\bar{\psi}}^{1,\dots,5}, \cancel{\bar{\eta}}^{1,2,3}, \bar{\phi}^{1,\dots,8}\}$$

So,

$$1 + b_1 + b_2 + b_3 = \{\phi^{1,\dots,8}\} = \zeta$$

The NAHE: The ABK Rules Obeyed

$$
\begin{aligned}
\mathbf{1}\cdot\mathbf{1} &= -12 \Rightarrow 0 \mod 8\\
\mathbf{S}\cdot\mathbf{S} &= +4 \Rightarrow 0 \mod 8\\
\mathbf{b_1}\cdot\mathbf{b_1} &= -4 \Rightarrow 0 \mod 8\\
\mathbf{b_2}\cdot\mathbf{b_2} &= -4 \Rightarrow 0 \mod 8\\
\mathbf{b_3}\cdot\mathbf{b_3} &= -4 \Rightarrow 0 \mod 8\\
\mathbf{1}\cdot\mathbf{S} &= +4 \Rightarrow 0 \mod 4\\
\mathbf{1}\cdot\mathbf{b_1} &= -4 \Rightarrow 0 \mod 4\\
\mathbf{1}\cdot\mathbf{b_2} &= -4 \Rightarrow 0 \mod 4\\
\mathbf{1}\cdot\mathbf{b_3} &= -4 \Rightarrow 0 \mod 4
\end{aligned}
$$

The NAHE: The ABK Rules Obeyed

$$\begin{aligned}
\mathbf{S}\cdot\mathbf{b_1} &= +2 \Rightarrow 0 \mod 4\\
\mathbf{S}\cdot\mathbf{b_2} &= +2 \Rightarrow 0 \mod 4\\
\mathbf{S}\cdot\mathbf{b_3} &= +2 \Rightarrow 0 \mod 4\\
\mathbf{b_1}\cdot\mathbf{b_2} &= -4 \Rightarrow 0 \mod 4\\
\mathbf{b_1}\cdot\mathbf{b_3} &= -4 \Rightarrow 0 \mod 4\\
\mathbf{b_2}\cdot\mathbf{b_3} &= -4 \Rightarrow 0 \mod 4
\end{aligned}$$

Summarise As

$$\mathbb{1} \cdot \mathbb{1} = -12$$
$$\mathbb{1} \cdot \mathbf{b_1} = -4$$
$$\mathbf{S} \cdot \mathbf{S} = 4$$
$$\mathbf{S} \cdot \mathbb{1} = 4$$
$$\mathbf{S} \cdot \mathbf{b_1} = 2$$
$$\mathbf{b_i} \cdot \mathbb{1} = -4, \;\; i = j = 1, 2, 3$$
$$\mathbf{b_i} \cdot \mathbf{S} = 2, \;\; i = j = 1, 2, 3$$
$$\mathbf{b_i} \cdot \mathbf{b_j} = -4, \;\; i = j = 1, 2, 3$$

The NAHE: The Basis $\{\mathbb{1}\}$
The Non-Supersymmetric Case

$$\Xi = \{NS, 1\}$$

The GSO coefficients in this case is

$$c\binom{NS}{NS} = 1, c\binom{NS}{\mathbb{1}} = -1, c\binom{\mathbb{1}}{NS} = -1, c\binom{\mathbb{1}}{\mathbb{1}} = -1$$

By the Virasoro level-matching condition the sector $\mathbb{1}$ does contain any massless states. The only sector containing the massless states is the Neveu-Schwarz sector.

The NAHE: The Basis $\{\mathbb{1}, \mathbf{S}\}$
SUSY

$$\Xi = \{NS, 1+S, 1, S\}$$

$$\begin{array}{c} \\ \mathbb{1} \\ \mathbf{S} \end{array} \begin{array}{c} \begin{array}{cc} \mathbb{1} & \mathbf{S} \end{array} \\ \begin{pmatrix} \pm 1 & 1 \\ 1 & 1 \end{pmatrix} \end{array}$$

The NAHE: The Basis $\{\mathbb{1}, \mathbf{S}, \mathbf{b_1}\}$

$$\Xi = \{NS, 1 + S, 1 + b_1, S + b_1, 1 + S + b_1, 1, S, b_1\}$$

The NAHE: The Basis $\{\mathbb{1}, \mathbf{S}, \mathbf{b_1}, \mathbf{b_2}\}$

$$
\begin{aligned}
\Xi \quad = \quad & \{NS, 1+S, 1+b_1, 1+b_2, S+b_1, S+b_2, b_1+b_2, 1+S+b_1, \\
& 1+S+b_2, 1+b_1+b_2, S+b_1+b_2, \\
& 1+S+b_1+b_2, 1, S, b_1, b_2\}
\end{aligned}
$$

The NAHE: The Basis $\{\mathbb{1}, \mathbf{S}, \mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}\}$

$$
\begin{aligned}
\Xi \; = \; & \{NS, 1+S, 1+b_1, 1+b_2, 1+b_3, S+b_1, S+b_2, S+b_3, \\
& b_1+b_2, b_1+b_3, b_2+b_3, 1+S+b_1, 1+S+b_2, 1+S+b_3, \\
& 1+b_1+b_2, 1+b_1+b_3, 1+b_2+b_3, S+b_1+b_2, S+b_1+b_3, \\
& S+b_2+b_3, b_1+b_2+b_3, 1+S+b_1+b_2, 1+S+b_1+b_3, \\
& 1+S+b_2+b_3, \zeta = 1+b_1+b_2+b_3, \\
& S+b_1+b_2+b_3, 1+S+b_1+b_2+b_3, 1, S, b_1, b_2, b_3\}
\end{aligned}
$$

The NAHE: ζ

$$\zeta = 1 + b_1 + b_2 + b_3$$

enhances the $SO(16) \to E_8$ as it contains

$$\left[\binom{8}{0} + \binom{8}{2} + \binom{8}{4} + \binom{8}{6} + \binom{8}{8}\right] = 128 \text{ states.}$$

The adjoint of $SO(16)$ has 120 states.

$$120 + 128 \Rightarrow \dim E_8$$

The NAHE: The Gauge Group

$$SO(44)$$
$$\downarrow$$
$$SO(10) \times E_8 \times SO(6)^3$$

with

$$N = 4$$
$$\downarrow$$
$$N = 2$$
$$\downarrow$$
$$N = 1$$

The Various $SO(10)$ Breakings

$$SO(10) \xrightarrow{\alpha+\beta} SU(5) \times U(1)$$

$$SO(10) \xrightarrow{\alpha} SO(6) \times SO(4) \xrightarrow{\beta} SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L$$

H. Sonmez (World Expert) $\longrightarrow SU(5) \times U(1)$

$$SO(10) \xrightarrow{\alpha+\beta+\gamma} SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R$$

Je Ne Sais Pas

THANK YOU!!!