

# Exotica and Discreteness in the Classification of String Spectra

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- Heterotic String Phenomenology
- Free Fermionic Construction
- $SO(10)$  GUTs
- Flipped  $SU(5)$  Models
- Classification
- Recent and Future Work

## Free Fermionic Construction

- 4D Theory
- $N = 1$  Supersymmetry
- 3 Generation Standard Model Fermions
- $SO(10)$  GUTs
- Absence of exotic states

# Free Fermionic Construction

## Properties

- Conformally invariance
- Decoupling left and right moving modes
- $D = 4$  theory

## Result

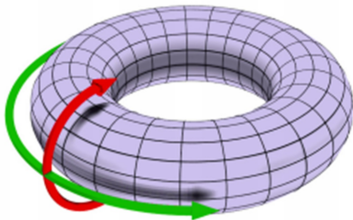
- $C_L = -26 + 11 + D + \frac{D}{2} + \frac{N_{f_L}}{2} = 0$   
 $\implies$  18 left-moving real fermions
- $C_R = 0$   
 $\implies$  44 right-moving real fermions

# Free Fermionic Construction

- Partition function is used to include all physical states

$$Z = \sum_{\alpha, \beta} c \begin{pmatrix} \alpha \\ \beta \end{pmatrix} Z [\alpha, \beta]$$

- Taking the one-loop partition function transforms the worldsheet into a torus.



# Free Fermionic Construction

$$\alpha = \left\{ \psi^{1,2}, \chi^i, y^i, \omega^i | \bar{y}^i, \bar{\omega}^i, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\dots,8} \right\}$$

Where  $i = 1, \dots, 6$

- Left-movers

- $X_L^\mu$ ,  $\mu = 1, 2$  2 transverse coordinates
- $\psi_L^\mu$ ,  $\mu = 1, 2$  The fermionic partners
- $\Omega^j$ ,  $j = 1, \dots, 18$  18 internal real fermions

- Right-movers

- $X_R^\mu$ ,  $\mu = 1, 2$  2 transverse coordinates
- $\bar{\Omega}^j$ ,  $j = 1, \dots, 44$  44 internal real fermions

# Free Fermionic Construction

- ABK Rules

- $\sum_i m_i b_i = 0$
- $N_{ij} \cdot b_i \cdot b_j = 0 \pmod{4}$
- $N_j \cdot b_i \cdot b_j = 0 \pmod{8}$
- $1 \in \Xi$
- Even number of fermions

- One-Loop Phases

- $C \begin{pmatrix} b_i \\ b_j \end{pmatrix} = \pm 1$  or  $\pm i$

- GSO Projection

- $e^{i\pi b_i \cdot F_\alpha} |s\rangle_\alpha = \delta_\alpha C \begin{pmatrix} \alpha \\ b_i \end{pmatrix}^* |s\rangle_\alpha$

- Virasoro Condition

- $M_L^2 = -\frac{1}{2} + \frac{\alpha_L^2}{8} + \sum v_L = -1 + \frac{\alpha_R^2}{8} + \sum v_R = M_R^2$

[I. Antoniadis et al, Nuclear Physics B (1987) 78-108]

## Basis Vectors

- $v_1 = 1 = \{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$
- $v_2 = S = \{\psi^\mu, \chi^{1,\dots,6}\}$
- $v_{2+i} = e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\} \quad i = 1, \dots, 6$
- $v_9 = b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}$
- $v_{10} = b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}$
- $v_{11} = z_1 = \{\bar{\phi}^{1,\dots,4}\}$
- $v_{12} = z_2 = \{\bar{\phi}^{5,\dots,8}\}$

## Gauge Group

$$SO(10) \times U(1)_1 \times U(1)_2 \times U(1)_3 \times SO(8)_1 \times SO(8)_2$$



# $SO(10)$ Breaking Basis Vectors

## Pati-Salam Models

- $v_{13} = \alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}$

## Flipped $SU(5)$ Models

- $v_{12} = \alpha = \{\bar{\eta}^{1,2,3} = \frac{1}{2}, \bar{\psi}^{1,\dots,5} = \frac{1}{2}, \bar{\phi}^{1,\dots,4} = \frac{1}{2}, \bar{\phi}^5 = 1\}$

## $SU(4) \times SU(2) \times U(1)$ Models

- $v_{13} = \alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}$

- $v_{14} = \beta = \{\bar{\psi}^{4,5} = \frac{1}{2}, \bar{\phi}^{1,\dots,6} = \frac{1}{2}\}$

## Standard-Like Models

- $v_{12} = \alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}$

- $v_{13} = \beta = \{\bar{\eta}^{1,2,3} = \frac{1}{2}, \bar{\psi}^{1,\dots,5} = \frac{1}{2}, \bar{\phi}^{1,\dots,4} = \frac{1}{2}, \bar{\phi}^5 = 1\}$

# Flipped $SU(5)$ Models

There are 3 basis vectors that can break the  $SO(10)$  gauge group giving rise to the flipped  $SU(5)$  models. The possible choices of  $\alpha$  are then given by:

- $\alpha_1 = \{\overline{\eta}^{1,2,3} = \frac{1}{2}, \overline{\psi}^{1,\dots,5} = \frac{1}{2}, \overline{\phi}^{1,\dots,4} = \frac{1}{2}, \overline{\phi}^5 = 1\}$
- $\alpha_2 = \{\overline{\eta}^{1,2,3} = \frac{1}{2}, \overline{\psi}^{1,\dots,5} = \frac{1}{2}, \overline{\phi}^{1,\dots,8} = \frac{1}{2}\}$
- $\alpha_3 = \{\overline{\eta}^{1,2,3} = \frac{1}{2}, \overline{\psi}^{1,\dots,5} = \frac{1}{2}, \overline{\phi}^{1,2} = \frac{1}{2}, \overline{\phi}^{5,6} = \frac{1}{2}, \overline{\phi}^8 = 1\}$

The matter content are encoded in the  $\mathbf{16}$  and  $\overline{\mathbf{16}}$  representations of  $SO(10)$  decomposed under  $SU(5) \times U(1)$

$$\mathbf{16} = (\overline{\mathbf{5}}, -\frac{3}{2}) + (\mathbf{10}, +\frac{1}{2}) + (\mathbf{1}, +\frac{5}{2}),$$

$$\overline{\mathbf{16}} = (\mathbf{5}, +\frac{3}{2}) + (\overline{\mathbf{10}}, -\frac{1}{2}) + (\mathbf{1}, -\frac{5}{2}).$$

# The Standard Model Particles

We can decompose the flipped  $SU(5)$  representation under  $SU(3) \times SU(2) \times U(1)$  as

$$\begin{aligned}(\bar{\mathbf{5}}, -\frac{3}{2}) &= (\bar{\mathbf{3}}, 1, -\frac{2}{3})_{u^c} + (1, 2, -\frac{1}{2})_L, \\(\mathbf{10}, +\frac{1}{2}) &= (3, 2, +\frac{1}{6})_Q + (\bar{\mathbf{3}}, 1, +\frac{1}{3})_{d^c} + (1, 1, 0)_{\nu^c}, \\(\mathbf{1}, +\frac{5}{2}) &= (1, 1, +1)_{e^c},\end{aligned}$$

# GGSO Coefficients

$$(v_i|v_j) =$$

	S	e <sub>1</sub>	e <sub>2</sub>	e <sub>3</sub>	e <sub>4</sub>	e <sub>5</sub>	e <sub>6</sub>	b <sub>1</sub>	b <sub>2</sub>	z <sub>1</sub>	z <sub>2</sub>	α
S	1	1	1	1	1	1	1	1	1	1	1	1
e <sub>1</sub>	1	0	0	1	1	1	0	0	0	1	1	1
e <sub>2</sub>	1	0	0	1	1	1	0	0	0	1	1	0
e <sub>3</sub>	1	1	1	0	0	1	0	0	0	1	1	0
e <sub>4</sub>	1	1	1	0	0	0	0	0	1	0	0	1
e <sub>5</sub>	1	1	1	1	0	0	0	0	1	0	1	0
e <sub>6</sub>	1	0	0	0	0	0	0	1	1	0	0	1
b <sub>1</sub>	0	0	0	0	0	0	1	1	0	1	1	1/2
b <sub>2</sub>	0	0	0	0	1	1	1	0	1	0	1	-1/2
z <sub>1</sub>	1	1	1	1	0	0	0	1	0	0	0	0
z <sub>2</sub>	1	1	1	1	0	1	0	1	1	0	0	1/2
α	1	1	0	0	1	0	1	0	1	1	1	1

Where  $C\left(\begin{smallmatrix} v_i \\ v_j \end{smallmatrix}\right) = e^{i\pi(v_i|v_j)}$   $(b_i|b_j) \in \{-\frac{1}{2}, 0, \frac{1}{2}, 1\}$

# GGSO Coefficients

$$\begin{array}{c}
 S \\
 e_1 \\
 e_2 \\
 e_3 \\
 e_4 \\
 e_5 \\
 e_6 \\
 b_1 \\
 b_2 \\
 z_1 \\
 z_2 \\
 \alpha
 \end{array}
 \begin{pmatrix}
 S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & b_1 & b_2 & z_1 & z_2 & \alpha \\
 & & 0_1 & 1_2 & 1_3 & 1_4 & 0_5 & 0_6 & & 1_7 & & 1_8 \\
 & & & 1_9 & 1_{10} & 1_{11} & 0_{12} & 0_{13} & & 1_{14} & & 0_{15} \\
 & & & & 0_{16} & 1_{17} & 0_{18} & & 0_{19} & 1_{20} & & 0_{21} \\
 & & & & & 0_{22} & 0_{23} & & 1_{24} & 0_{25} & & 1_{26} \\
 & & & & & & 0_{27} & 0_{28} & 1_{29} & 0_{30} & & 0_{31} \\
 & & & & & & & 1_{32} & 1_{33} & 0_{34} & & 1_{35} \\
 & & & & & & & & 0_{36} & & 1_{37} & 1/2_{38} \\
 & & & & & & & & & 0_{39} & 1_{40} & -1/2_{41} \\
 & & & & & & & & & & 0_{42} & 0_{43} \\
 & & & & & & & & & & & 1/2_{44}
 \end{pmatrix}$$

Depending on the choices of the projection coefficients, extra gauge bosons may arise. These arise with any linear combination of  $z_1$ ,  $z_2$  and  $\alpha$ , which are massless. Such as

$$\mathbf{G} = \left\{ \begin{array}{cccc} z_1, & z_2, & z_1 + z_2, & \alpha, \\ \alpha + z_1, & \alpha + z_2, & \alpha + z_1 + z_2, & x \end{array} \right\}$$

Where

$$x = 2\alpha + z_1 = \{\bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}\}$$

# Observable Gauge Group Enhancements

$x$  is a sector which can enlarge the observable gauge group. Enhancement takes place when the following conditions are satisfied

Sector Condition	
$(z_1 + 2\alpha e_i) = (z_1 + 2\alpha z_k) = 0$	
Enhancement Condition	Resulting Enhancement
$(z_1 + 2\alpha \alpha) = (z_1 + 2\alpha b_2)$	$SU(5)_{obs} \times U(1)_5 \times U(1)_1 \times U(1)_2 \times U(1)_3$ $\longrightarrow SU(6) \times SU(2) \times U(1)^2$
$(z_1 + 2\alpha \alpha) \neq (z_1 + 2\alpha b_2)$	$SU(5)_{obs} \times U(1)_5 \times U(1)_1 \times U(1)_2 \times U(1)_3$ $\longrightarrow SO(10) \times U(1)^3$

The pre-stated conditions hold for all  $i = 1, \dots, 6$ .



The chiral matter spectrum arises from the twisted sectors. The chiral spinorial representations of the observable  $SU(5) \times U(1)$  arise from the sectors

$$\begin{aligned} B_{pqrs}^{(1)} &= S + b_1 + pe_3 + qe_4 + re_5 + se_6, \\ &= \{\psi^\mu, \chi^{12}, (1-p)y^3\bar{y}^3, p\omega^3\bar{\omega}^3, (1-q)y^4\bar{y}^4, q\omega^4\bar{\omega}^4, \\ &\quad (1-r)y^5\bar{y}^5, r\omega^5\bar{\omega}^5, (1-s)y^6\bar{y}^6, s\omega^6\bar{\omega}^6, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \end{aligned}$$

$$B_{pqrs}^{(2)} = S + b_2 + pe_1 + qe_2 + re_5 + se_6,$$

$$B_{pqrs}^{(3)} = S + b_3 + pe_1 + qe_2 + re_3 + se_4,$$

Where  $p, q, r, s = 0, 1$  and  $b_3 = b_1 + b_2 + 2\alpha + z_1$ .

The states in the sector  $B_{pqrs}^{(1)}$  can be projected out of the spectrum by the GGSO projection of the vectors  $e_1$ ,  $e_2$ ,  $z_1$  and  $z_2$ . Similarly for all sectors, we can define a projector  $P$  such that the states survive when  $P = 1$  and are projected out when  $P = 0$ :

$$P_{pqrs}^{(1)} = \frac{1}{16} \left( 1 - C \left( \begin{matrix} e_1 \\ B_{pqrs}^{(1)} \end{matrix} \right) \right) \cdot \left( 1 - C \left( \begin{matrix} e_2 \\ B_{pqrs}^{(1)} \end{matrix} \right) \right) \cdot \left( 1 - C \left( \begin{matrix} z_1 \\ B_{pqrs}^{(1)} \end{matrix} \right) \right) \cdot \left( 1 - C \left( \begin{matrix} z_2 \\ B_{pqrs}^{(1)} \end{matrix} \right) \right)$$

$$P_{pqrs}^{(2)} = \frac{1}{16} \left( 1 - C \left( \begin{matrix} e_3 \\ B_{pqrs}^{(2)} \end{matrix} \right) \right) \cdot \left( 1 - C \left( \begin{matrix} e_4 \\ B_{pqrs}^{(2)} \end{matrix} \right) \right) \cdot \left( 1 - C \left( \begin{matrix} z_1 \\ B_{pqrs}^{(2)} \end{matrix} \right) \right) \cdot \left( 1 - C \left( \begin{matrix} z_2 \\ B_{pqrs}^{(2)} \end{matrix} \right) \right)$$

$$P_{pqrs}^{(3)} = \frac{1}{16} \left( 1 - C \left( \begin{matrix} e_5 \\ B_{pqrs}^{(3)} \end{matrix} \right) \right) \cdot \left( 1 - C \left( \begin{matrix} e_6 \\ B_{pqrs}^{(3)} \end{matrix} \right) \right) \cdot \left( 1 - C \left( \begin{matrix} z_1 \\ B_{pqrs}^{(3)} \end{matrix} \right) \right) \cdot \left( 1 - C \left( \begin{matrix} z_2 \\ B_{pqrs}^{(3)} \end{matrix} \right) \right)$$

# Projectors

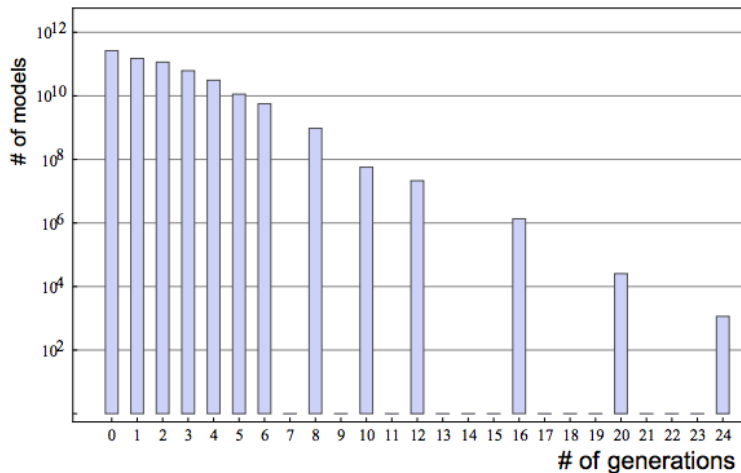
These projectors can be expressed as a system of linear equations with  $p$ ,  $q$ ,  $r$  and  $s$  as unknowns. The solutions of a specific system of equations yield the different combinations of  $p$ ,  $q$ ,  $r$  and  $s$  for which sectors survive the GSO projections.

$$\begin{pmatrix} (e_1|e_3) & (e_1|e_4) & (e_1|e_5) & (e_1|e_6) \\ (e_2|e_3) & (e_2|e_4) & (e_2|e_5) & (e_2|e_6) \\ (z_1|e_3) & (z_1|e_4) & (z_1|e_5) & (z_1|e_6) \\ (z_2|e_3) & (z_2|e_4) & (z_2|e_5) & (z_2|e_6) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_1|b_1) \\ (e_2|b_1) \\ (z_1|b_1) \\ (z_2|b_1) \end{pmatrix},$$

$$\begin{pmatrix} (e_3|e_1) & (e_3|e_2) & (e_3|e_5) & (e_3|e_6) \\ (e_4|e_1) & (e_4|e_2) & (e_4|e_5) & (e_4|e_6) \\ (z_1|e_1) & (z_1|e_2) & (z_1|e_5) & (z_1|e_6) \\ (z_2|e_1) & (z_2|e_2) & (z_2|e_5) & (z_2|e_6) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_3|b_2) \\ (e_4|b_2) \\ (z_1|b_2) \\ (z_2|b_2) \end{pmatrix},$$

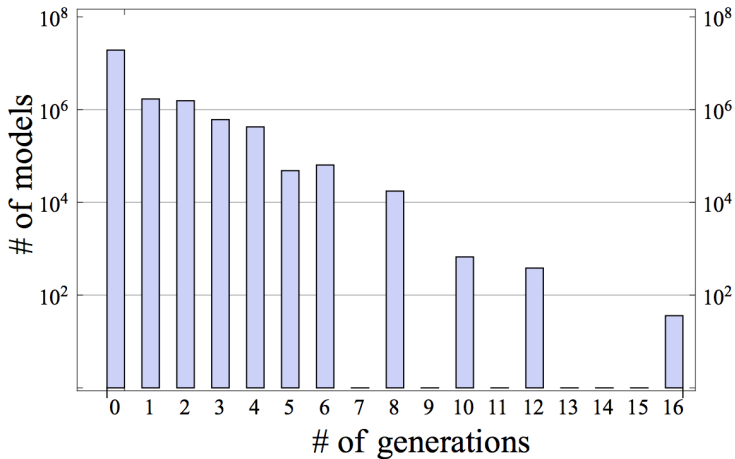
$$\begin{pmatrix} (e_5|e_1) & (e_5|e_2) & (e_5|e_3) & (e_5|e_4) \\ (e_6|e_1) & (e_6|e_2) & (e_6|e_3) & (e_6|e_4) \\ (z_1|e_1) & (z_1|e_2) & (z_1|e_3) & (z_1|e_4) \\ (z_2|e_1) & (z_2|e_2) & (z_2|e_3) & (z_2|e_4) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_5|b_3) \\ (e_6|b_3) \\ (z_1|b_3) \\ (z_2|b_3) \end{pmatrix}.$$

# SO(10) Classification



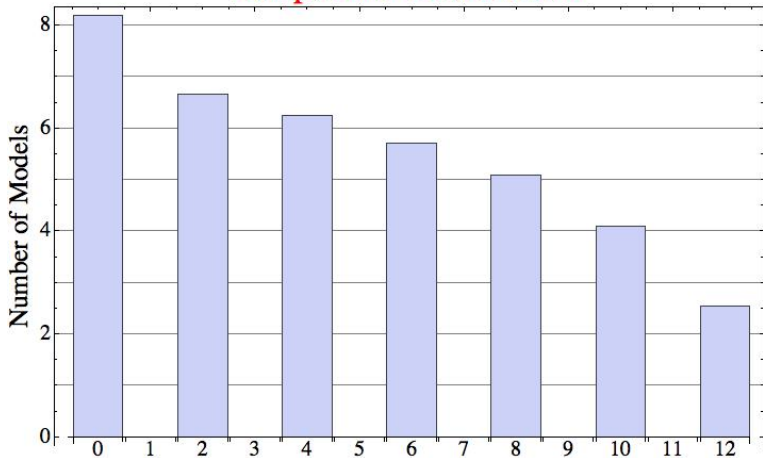
[Figure 1: John Rizos, arXiv:1105.1243]

# Pati-Salam Classification

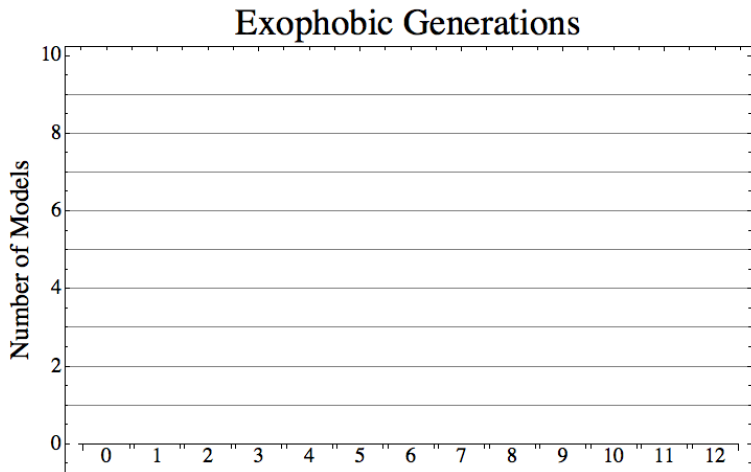


[Figure 3: B.Assel et al, arXiv:1007.2268]

## Exophobic Generations



[Figure 2: H. Sonmez et al, arXiv:1403.4107]

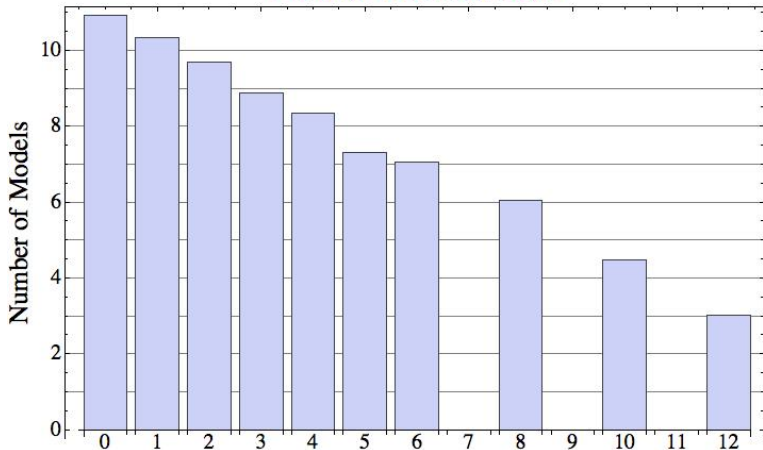


[H. Sonmez et al, arXiv:1412.2839]





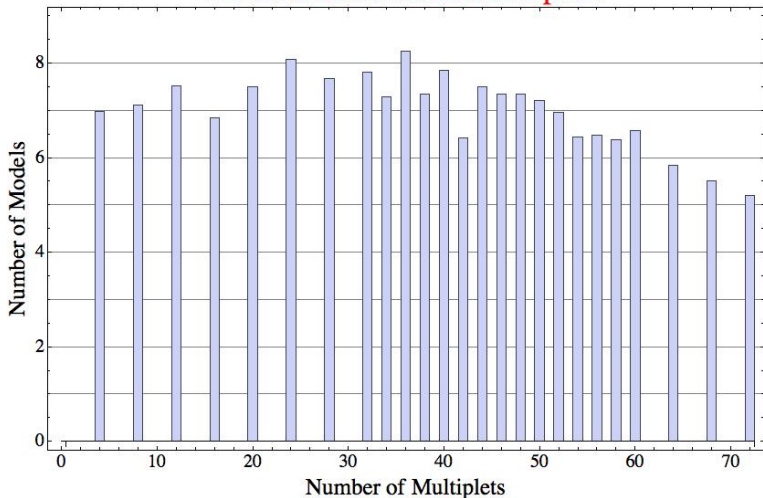
## Exotic Generations



[Figure 1: H. Sonmez et al, arXiv:1403.4107]

# Flipped $SU(5)$ Classification

## 3 Generation Exotic Multiplets



[Figure 3: H. Sonmez et al, arXiv:1403.4107]

# Flipped $SU(5)$ Classification

	Constraints	Total models in sample	Probability	Estimated number of models in class
	No Constraints	1000000000000	1	$1.76 \times 10^{13}$
(1)	+ No Enhancements	762269298719	$7.62 \times 10^{-1}$	$1.34 \times 10^{13}$
(2)	+ Anomaly Free Flipped $SU(5)$	139544182312	$1.40 \times 10^{-1}$	$2.45 \times 10^{12}$
(3)	+ 3 Generations	738045321	$7.38 \times 10^{-4}$	$1.30 \times 10^{10}$
(4a)	+ SM Light Higgs	706396035	$7.06 \times 10^{-4}$	$1.24 \times 10^{10}$
(4b)	+ Flipped $SU(5)$ Heavy Higgs	46470138	$4.65 \times 10^{-5}$	$8.18 \times 10^8$
(5)	+ SM Light Higgs + & Heavy Higgs	43624911	$4.36 \times 10^{-5}$	$7.67 \times 10^8$
(6a)	+ Minimal Flipped $SU(5)$ Heavy Higgs	42310396	$4.23 \times 10^{-5}$	$7.44 \times 10^8$
(6b)	+ Minimal SM Light Higgs	25333216	$2.53 \times 10^{-5}$	$4.46 \times 10^8$
(7)	+ Minimal Flipped $SU(5)$ Heavy Higgs + & Minimal SM Light Higgs	24636896	$2.46 \times 10^{-5}$	$4.33 \times 10^8$
(8)	+ Minimal Exotic States	1218684	$1.22 \times 10^{-6}$	$2.14 \times 10^7$

[Figure 3: H. Sonmez et al, arXiv:1403.4107]

Recent and Future Work:

- 1  $SU(4) \times SU(2)_L \times U(1)_L$
- 2  $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L$
- 3  $SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R$
- 4  $SU(5) \times U(1)$
- 5 Non-Supersymmetric Heterotic String Vacua

THANK YOU