# Nernst Branes from special geometry

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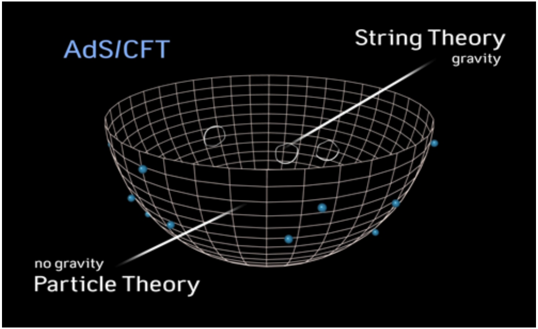
# Outline

- Holographic Motivation
- Real formulation of special geometry
- Constructing Nernst branes
- Interpretation
- Conclusion and Outlook

# $AdS_{d+1}/CFT_d$




- asymptotically AdS gravity in bulk $\longleftrightarrow$ CFT on boundary
- strong/weak coupling duality
- explore previously inaccessible systems e.g. AdS/CMT

# AdS/CMT

black objects in bulk $\longleftrightarrow$ thermal ensemble in field theory with same thermodynamic properties $(T, S, \mu, \ldots)$

- CMT obeys all thermodynamic laws.
- There is a well established correspondence between laws of thermodynamics and laws of black hole mechanics.
- We need to build black objects that satisfy **all** of these.

# Nernst Law/3$^{rd}$ law of thermodynamics

- All black objects seem to satisfy the $0^{th}, 1^{st}$ and $2^{nd}$ laws.
- There are several different forms of third law.
- We follow strictest definition (unique ground state):

$$S \xrightarrow{T \longrightarrow 0} 0 \quad \text{holding other parameters fixed}$$

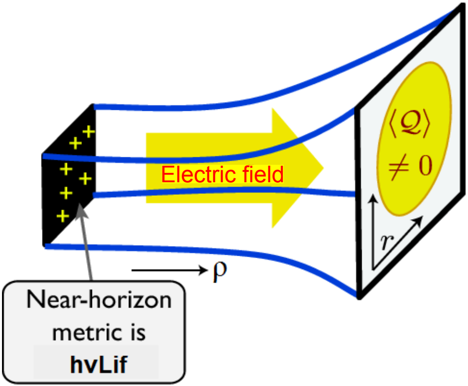
- Not always true e.g. RN black holes/branes have large $S(T=0) \neq 0$ indicating there isn't a unique ground state.
- Explained by microstate counting of D-branes or by stringy higher curvature corrections for certain BPS BHs
- Are there gravitational systems with $S(T=0)=0$?
- There do exist *small black holes* with $S(T=0)=0$ but ...

# Why branes?

- $S \xrightarrow{T \to 0} 0$ means vanishing horizon area in extremal limit.
- These satisfy Nernst law but $A \xrightarrow{T \longrightarrow 0} 0$ means $r_H \longrightarrow 0$
- SUGRA approx valid when $\mathcal{R}_H < \mathcal{R}_P$.
- $S^{d-2}$ horizon topology $\Rightarrow \mathcal{R}_H \sim \frac{1}{r_H^2}$.
- $\Rightarrow \mathcal{R}_H \xrightarrow{T \longrightarrow 0} \infty$
- Small black holes unsuitable for SUGRA analysis.
- Natural to turn to black branes with Ricci-flat horizons.

# Why gSUGRA?

- Without fluxes 4d black objects have $S^2$ horizon topology.
- Turn on FI gauging to produce branes i.e. use gSUGRA.
- c.f. fluxes along internal manifold



# Nernst branes in gSUGRA

**Goal:** Systematically construct a family of non-extremal black branes in 4d, $\mathcal{N} = 2$ gSUGRA s.t. $s \xrightarrow{T \longrightarrow 0} 0$ i.e. *Nernst branes*.

**Why non-extremal?**

- Extremal Nernst branes turn out to not be completely regular suggesting breakdown of effective theory.
- Find non-extremal solns and study them in near extremal limit to address this.
- Want completely analytic results for this. Literature has mixture of analytic/numerical.

# What's been done?

- Barisch, Cardoso, Haack, Nampuri, Obers 1108.0296
  - Use 1st order flow eqns to construct extremal 4d Nernst brane i.e. a black brane with $s(T=0)=0$.
  - Don't construct non-extremal branes.
- Goldstein, Nampuri, Véliz-Osorio 1406.2937
  - Obtain extremal Nernst brane in 5d.
  - Provide algorithm to deform extrmal soln into corresponding "hot" (non-extremal) soln.
- Dempster, DE, Mohaupt 1501.07863
  - Using real formulation of special geometry and dimensional reduction, we make optimal use of EM duality and solve full 2nd order EoMs to obtain 4d non-extremal solns.
  - Don't restrict to particular model: class of very special prepotentials.
  - Technique not restricted to models with symmetric target spaces.

# Gauged SUGRA

- Consistent duality requires bulk gravity to have well-defined UV completion i.e. embedding in string theory.
- gSUGRA is LEEFT arising through flux compactifications on $K3 \times T^2$ or $CY_3$.
- 4d bosonic Lagrangian of $n$ VMs coupled to $\mathcal{N} = 2$ $U(1) \subset SU(2)_R$ gSUGRA is

$$e_4^{-1}\mathcal{L}_4 = -\frac{1}{2}YR_4 - g_{IJ}\partial_{\hat{\mu}}X^I\partial^{\hat{\mu}}\bar{X}^J + \frac{1}{4}\mathcal{I}_{IJ}F^I_{\hat{\mu}\hat{\nu}}F^{J|\hat{\mu}\hat{\nu}} + \frac{1}{4}\mathcal{R}_{IJ}F^I_{\hat{\mu}\hat{\nu}}\tilde{F}^{J|\hat{\mu}\hat{\nu}} - V(X,\bar{X})$$

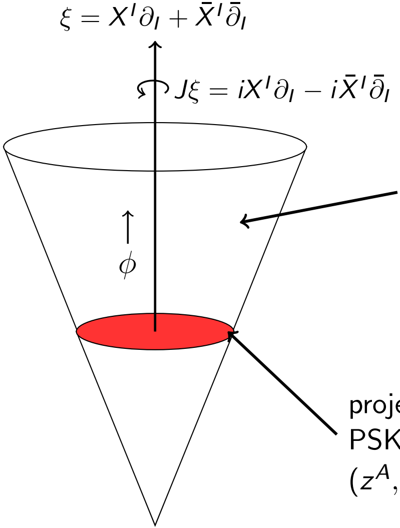
$$V(X,\bar{X}) = \partial^I W \partial_I \bar{W} - 2\kappa^2|W|^2, \qquad W = 2\left(g^I F_I - g_I X^I\right).$$

$$\hat{\mu} = 0,\ldots,3, \qquad I,J = 0,\ldots,n, \qquad F(X)\text{hom. deg. } 2.$$

- Work on 'big moduli space' with $X^I, I = 0,\ldots,n$ rather than physical $z^A, A = 1,\ldots,n$.
- Extra cx d.o.f. compensated for by $\mathbb{C}^*$ gauge symmetry.
- $\#$ scalars $= \#$ gauge fields $\Rightarrow$ symplectic covariance.

# Target Manifolds - visualise additional real d.o.f.

$$\mathbb{C}^* = \mathbb{R}^{>0} \cdot U(1)$$




conic affine special Kähler, CASK
$(X^I, N_{IJ})$

projective special Kähler,
PSK = CASK/$\mathbb{C}^*$
$(z^A, g_{A\bar{B}})$ with $z^A = \frac{X^A}{X^0}$

# Gauge Fixing

Gauge fix to go from superconformal theory to physical theory.

How do we do this?

- D-gauge fixes dilatations:

$$Y = -i\left(X^I \bar{F}_I - \bar{X}^I F_I\right) = \kappa^{-2}$$

$$\Rightarrow -\frac{1}{2} Y R_4 = -\frac{1}{2\kappa^2} R_4 .$$

- $U(1)$ transformations fixed by $\operatorname{Im}\left(X^0\right) = 0$.

  We postpone this to retain symplectic covariance and work in a $U(1)$ principal bundle instead over PSK base.

# Real coordinates 1

- Story so far has been using complex coords.
- We use **real formulation of special Kähler geometry**.
  [Freed: hep-th/9712042]
  [Alekseevsky, Cortés, Devchand: hep-th/9910091]
- Already been used to great success for building solns to
  - ungauged SUGRA coupled to VMs
    [Mohaupt, Vaughan: hep-th/1112 : 2876]
    [DE,Mohaupt,Vaughan: hep-th/1408.0923]
  - gauged SUGRA coupled to VMs
    [Klemm,Vaughan: hep-th/1207.2679 & hep-th/1211.1618]
    [Gnecchi, Hristov, Klemm, Toldo, Vaughan: hep-th/1311.1795]
- **Advantage:** Symplectic covariance + tensorial behaviour
  $\Rightarrow$ everything transforms linearly.

# Real coordinates 2

- $X^I = x^I + iu^I, \qquad F_I = y_I + iv_I$
- $q^a = \text{Re}\ \left(X^I, F_I\right)^T = \left(x^I, y_I\right)^T, \quad a = 0, \ldots, 2n+1.$
  form real coordinate system on CASK (retain $\mathbb{C}^*$ action over PSK).
- Prepotential, $F(X) \xrightarrow{\text{Legendre transf.}}$ Hesse potential, $H(q^a)$
- Convenient to introduce **dual coordinates**:
  $q_a = H_a = \frac{\partial H}{\partial q^a} = 2\text{Im}\ \left(F_I, -X^I\right)^T = \left(2v_I, -2u^I\right)^T$
- $H_{ab} = \frac{\partial^2 H}{\partial q^a \partial q^b}$ is real version of $N_{IJ}$ (CASK metric):
  $q^a = H^{ab} q_b$ and $q_a = H_{ab} q^b$
- Tensorial behaviour is natural $\Rightarrow$ simplifies calculations!

# Dimensional Reduction 1

- Seek stationary (actually static) brane solns allows dimensional reduction over timelike $S^1$.
- KK ansatz: $ds_4^2 = -e^{\phi}\left(dt + V_\mu dx^\mu\right)^2 + e^{-\phi} ds_3^2$ with $\phi$, $V$ the KK scalar and vector resp.
- Identify radial direction of cone with KK scalar.
- Promote radial direction of cone from gauge d.o.f. to physical d.o.f. by rescaling complex symplectic vector: $\left(Y^I, F_I(Y)\right)^T = e^{\frac{\phi}{2}}\left(X^I, F_I(X)\right)^T$
- Must redefine real symplectic vector: $q^a = \left(x^I, y_I\right)^T = \text{Re}\ \left(Y^I, F_I(Y)\right)^T$ (similar for $q_a$)
- D-gauge: $-i\left(X^I \bar{F}_I(X) - F_I(X)\bar{X}^I\right) = 1$ (with $\kappa = 1$) $\longrightarrow -2H = -i\left(Y^I \bar{F}_I(Y) - F_I(Y)\bar{Y}^I\right) = e^{\phi}$

# Dimensional Reduction 2

- At 3d level, we find additional scalars:
- $\hat{A}^I_{\hat{\mu}}(t,x)dx^{\hat{\mu}} = \xi^I dt + \left[A^I_\mu(x) + \xi^I V_\mu(x)\right] dx^\mu$
  $\Rightarrow \hat{A}^I(t,x) = \xi^I dt + \tilde{A}^I_\mu(x)dx^\mu$
- $\tilde{A}^I \overset{\star}{\longleftrightarrow} \tilde{\xi}_I \qquad V \overset{\star}{\longleftrightarrow} \tilde{\phi}$
- $\hat{q}^a = \left(\frac{1}{2}\xi^I, \frac{1}{2}\tilde{\xi}_I\right)^T$ with $\left(\partial_\mu \xi^I, \partial_\mu \tilde{\xi}_I\right)^T = \left(F^I_{\mu 0}, \tilde{G}_{I|\mu 0}\right)^T$
- There are $4n+5$ 3d scalars
- $U(1)$ bundle over $4n+4$ dimensional para-QK mfold.

# Model Constraints

- Focus on **very special models** that can be lifted to 5d.
- $F(Y) = \frac{f(Y^1,\ldots,Y^n)}{Y^0}$ $f$ hom. deg. 3 and real when evaluated on real fields.
- Also restrict to **purely imaginary** field config Re $(z^A) = 0$
- $z^A = \frac{Y^A}{Y^0} = \frac{x^A + iu^A}{x^0}$
- PI $\Rightarrow x^A = 0$ and must set $y_0 = 0$ for consistency.

$$q^a = \left(x^0, x^A; y_0, y_A\right)^T \xrightarrow{\text{PI}} q^a|_{\text{PI}} = \left(x^0, 0, \ldots, 0; 0, y_1, \ldots, y_n\right)^T$$

$$\Rightarrow q_a = \frac{1}{H}\left(-v_0, -v_A; u^0, u^A\right)^T \xrightarrow{\text{PI}} q_a|_{\text{PI}} = \frac{1}{H}\left(-v_0, 0, \ldots, 0; 0, u^1, \ldots, u^n\right)^T$$

- $q^a, q_a$ are symplectic vectors. Now only want to allow transformations by Stab$(PI) \subset$ Symp$(2n+2, \mathbb{R})$
- Natural to extend PI to $\partial_\mu \hat{q}^a$ and $g^a = \left(g^I, g_I\right)^T$.
- Greatly simplifies EoMs

# 3d Lagrangian

Reduce to 3d Euclidean theory and repackage d.o.f. using real coords. Then restrict to static and purely imaginary branes to find:

$$e_3^{-1}\mathcal{L}_3 = -\frac{1}{2}R_3 - \tilde{H}_{ab}\left(\partial_\mu q^a \partial_\mu q^b - \partial_\mu \hat{q}^a \partial_\mu \hat{q}^b - g^a g^b\right) + 4\left(g^a q_a\right)^2$$

$\tilde{H}_{ab}$ is modified metric on CASK:

$\tilde{H}_{ab} = \frac{\partial^2 \tilde{H}}{\partial q^a \partial q^b}$ with $\tilde{H} = -\frac{1}{2}\log\left(-2H\right)$

$$\tilde{H}^{ab}|_{\mathrm{PI}} = \left(\begin{array}{c|ccc|ccc} \tilde{H}^{00}(q_0) & & 0 & & & 0 & \\ \hline & * & \dots & * & & & \\ 0 & \vdots & \ddots & \vdots & & 0 & \\ & * & \dots & * & & & \\ \hline & & & & \tilde{H}^{n+2,n+2}(q_A) & \dots & \tilde{H}^{n+2,2n+1}(q_A) \\ 0 & & 0 & & \vdots & \ddots & \vdots \\ & & & & \tilde{H}^{2n+1,n+2}(q_A) & \dots & \tilde{H}^{2n+1,2n+1}(q_A) \end{array}\right)$$

# EoMs

Scalar equations of motion:

$$\nabla^2 \hat{q}_a = 0$$

$$\nabla^2 q_a + \frac{1}{2}\partial_a \tilde{H}^{bc}\left(\partial_\mu q_b \partial^\mu q_c - \partial_\mu \hat{q}_b \partial^\mu \hat{q}_c\right) - \frac{1}{2}\partial_a \tilde{H}_{bc} g^b g^c + 4\tilde{H}_{ab} g^b \left(g^c q_c\right) = 0$$

$$-\frac{1}{2}R_{3|\mu\nu} - \tilde{H}^{ab}\left(\partial_\mu q_a \partial_\nu q_b - \partial_\mu \hat{q}_a \partial_\nu \hat{q}_b\right) + g_{\mu\nu}\left(-\tilde{H}_{ab} g^a g^b + 4\left(g^a q_a\right)^2\right) = 0$$

**Goal:** solve these EoMs to find 3d instantons that we can lift back to regular 4d black branes.

# Electric Black Branes

- We want Nernst brane solutions supported by:
  - single electric charge, $Q_0$
  - electric fluxes $g_1, \ldots, g_n$
- In paper, we also discuss situation with single magnetic charge, $P^0$, and electric/magnetic fluxes.
- Leave thorough analysis of dyonic case to future work.

## Metric Components

For very special prepotentials, $F(Y) = \frac{f(Y^1,\ldots,Y^n)}{Y^0}$, we find

$$H = -\frac{1}{4}\left(-q_0 f(q_1,\ldots,q_n)\right)^{-\frac{1}{2}}$$

.
- PI config necessary to perform Legendre transformation and find explicit form of $H$.
- For general $f$, $\tilde{H}^{ab}$ is complicated.
- But since $q_0$ is decoupled by PI condition, we can compute

$$\tilde{H}^{00} = \frac{1}{4q_0^2}, \quad q^0 = -\frac{1}{4q_0}$$

- This turns out to be sufficient to find solutions valid for any $f$.

# Hamiltonian Constraint

3d metric ansatz: $ds_3^2 = e^{4\psi(\tau)} d\tau^2 + e^{2\psi(\tau)} \left(dx^2 + dy^2\right)$

where $\psi(\tau)$ is yet to be determined and $q_a, \hat{q}_a$ only depend on $\tau$.
N.B. $\tau$ is affine parameter for 'geodesics' (with potential) on pQK.

From above metric we can compute $R_{3|\mu\nu}$ and Einstein equations from $\mathcal{L}_3$ become

$$-\tilde{H}_{ab} g^a g^b + 4(q_a g^a)^2 - \frac{1}{2} e^{-4\psi} \ddot{\psi} = 0 \qquad \text{for } \mu = \nu \neq \tau$$

$$\tilde{H}^{ab} \left(\dot{q}_a \dot{q}_b - \dot{\hat{q}}_a \dot{\hat{q}}_b\right) = \dot{\psi}^2 - \frac{1}{2} \ddot{\psi} \qquad \text{for } \mu = \nu = \tau$$

$\tau\tau$ equation equivalent to Hamiltonian constraint.

# $\hat{q}_a$ EoM

$$\ddot{\hat{q}}_a = 0 \quad \Rightarrow \quad \dot{\hat{q}}_a = K_a$$

The consts $K_a$ are proportional to electric/magnetic charges i.e.

$$K_a = \left(-Q_I, P^I\right)^T$$

We only have a single electric charge:

$$\dot{\hat{q}}_0 = -Q_0, \qquad \dot{\hat{q}}_a = 0 \quad \forall a \neq 0$$

# $q_0$ EoM

Recall the $q_a$ EoM was

$$e^{-4\psi}\ddot{q}_a+\frac{1}{2}\partial_a\tilde{H}^{bc}e^{-4\psi}\left(\dot{q}_b\dot{q}_c-\dot{\hat{q}}_b\dot{\hat{q}}_c\right)-\frac{1}{2}\partial_a\tilde{H}_{bc}g^b g^c+4\tilde{H}_{ab}g^b(q_c g^c)=0$$

Because there is no magnetic flux $g^0=0$, the $q_0$ EoM decouples. Substituting $\dot{\hat{q}}_0=-Q_0$ gives

$$\ddot{q}_0-\frac{\dot{q}_0^2-Q_0^2}{q_0}=0$$

with the same solution as in ungauged case:

$$q_0(\tau)=\pm\frac{-Q_0}{B_0}\sinh\left(B_0\tau+B_0\frac{h_0}{Q_0}\right)\qquad B_0=\text{non-ext parameter}$$

with $B_0, h_0$ constants that satisfy $B_0\geq 0, \mathrm{sign}(h_0)=\mathrm{sign}(Q_0)$.

# $q_A$ EoM

These are the difficult eqns to solve:

$$e^{-4\psi}\ddot{q}_A + \tfrac{1}{2}e^{-4\psi}\sum_{B,C=1}^{n}\partial_A\tilde{H}^{BC}\dot{q}_B\dot{q}_C - \frac{1}{2}\sum_{B,C=1}^{n}(\partial_A\tilde{H}_{BC})g_B g_C + 4\sum_{B=1}^{n}\tilde{H}_{AB}g_B\left(\sum_{C=1}^{n}q_C g_C\right) = 0$$

Multiply by $q^A$ and use homogeneity to obtain:

$$e^{-4\psi}\sum_{A=1}^{n}q^A\ddot{q}_A + e^{-4\psi}\sum_{A,B=1}^{n}\tilde{H}^{AB}\dot{q}_A\dot{q}_B + \sum_{A,B=1}^{n}\tilde{H}_{AB}g_A g_B - 4\left(\sum_{A=1}^{n}q_A g_A\right)^2 = 0$$

Substituting the $\mu = \nu \neq \tau$ Einstein equation and integrating gives:

$$\sum_{A=1}^{n}q^A\dot{q}_A = \frac{1}{2}\dot{\psi} - \frac{1}{4}a_0 \quad \text{with } a_0 \text{ an integration constant.}$$

Then, since $\frac{d\tilde{H}}{d\tau} = \frac{\dot{q}_0}{4q_0} - \sum_{A=1}^{n}q^A\dot{q}_A$, we can substitute this and integrate:

$$\log\left(f(q_1,\ldots,q_n)\right) = -2\psi + a_0\tau + b_0 \quad \text{with } b_0 \text{ another integration const.}$$

Picture: $q_A$ are solns of top eqn constrained by above eqn and also $\tau\tau$ Einstein eqn:

$$\sum_{A,B=1}^{n}\tilde{H}^{AB}\dot{q}_A\dot{q}_B = \dot{\psi}^2 - \frac{1}{2}\ddot{\psi} - \frac{1}{4}B_0^2$$

# $q_A$ EoM

- We proceed by imposing $q_A(\tau) = \xi_A q(\tau)$
- This will force physical $z^A$ proportional to one another.
- There remain $n$ arbitrary electric flux parameters, $g_A$.

The two constraints from previous slide become:

$$3\left(\frac{\dot{q}}{q}\right)^2 = 4\dot{\psi}^2 - 2\ddot{\psi} - B_0^2, \qquad 3\left(\frac{\dot{q}}{q}\right) = -2\dot{\psi} + a_0$$

Combine these to get second order differential eqn:

$$\ddot{\psi} - \tfrac{4}{3}\dot{\psi}^2 - \tfrac{2}{3}a_0\dot{\psi} + \tfrac{1}{2}B_0^2 + \tfrac{1}{6}a_0^2 = 0$$

Let $y := \exp\left(-\frac{4}{3}\psi - \frac{1}{3}a_0\tau\right)$ then this is harmonic oscillator:

$$\ddot{y} - \omega^2 y = 0, \qquad \text{where} \qquad \omega^2 = \tfrac{2}{3}B_0^2 + \tfrac{1}{3}a_0^2$$

and solution is

$$\exp\left(-\tfrac{4}{3}\psi - \tfrac{1}{3}a_0\tau\right) = \tfrac{\alpha}{\omega}\sinh\left(\omega\tau + \omega\beta\right)$$ with $\alpha, \beta > 0$ integration consts.

This implies $e^{-4\psi} = \left(\frac{\alpha}{\omega}\right)^3 \sinh^3\left(\omega\tau + \omega\beta\right)e^{a_0\tau}$ [3d metric d.o.f.]

# $q_A$ EoM

- Differentiate the metric d.o.f. and substitute into second constraint.
- After some straightforward algebra we arrive at

$$q_A = \lambda_A e^{\frac{1}{2}a_0\tau} \left(\sinh\left(\omega\tau + \omega\beta\right)\right)^{\frac{1}{2}}$$

- Substituting this into original $q_A$ EoM, we find that $q_1 g_1 = \cdots = q_n g_n$ and EoM only satisfied if

$$\lambda_A = \pm\frac{3}{8ng_A}\left(\frac{\alpha^3}{\omega}\right)^{\frac{1}{2}}$$

- Final expression:

$$q_A = \pm\frac{3}{8ng_A}\left(\frac{\alpha^3}{\omega}\right)^{\frac{1}{2}} e^{\frac{1}{2}a_0\tau} \left(\sinh\left(\omega\tau + \omega\beta\right)\right)^{\frac{1}{2}}$$

# Regular Black Brane Solution

Black brane solution has metric

$$ds_4^2 = -e^{\phi} dt^2 + e^{-\phi+4\psi} d\tau^2 + e^{-\phi+2\psi} \left(dx^2 + dy^2\right)$$

where $\tau \to 0$ represents the asymptotic regime and $\tau = \infty$ is the event horizon.

The metric d.o.f. are

$$e^{-4\psi} = \left(\frac{1}{B_0}\right)^3 \sinh^3(B_0\tau) e^{B_0\tau},$$

$$e^{\phi} = -2H = \frac{1}{2}(-q_0)^{\frac{1}{2}} \left(f(q_1, \ldots, q_n)\right)^{-\frac{1}{2}},$$

with scalar fields given by

$$q_0 = \pm \frac{-Q_0}{B_0} \sinh\left(B_0\tau + B_0 \frac{h_0}{Q_0}\right), \qquad q_A = \pm \frac{3}{8ng_A} \left(\frac{1}{B_0}\right)^{\frac{1}{2}} e^{\frac{1}{2}B_0\tau} \left(\sinh(B_0\tau)\right)^{\frac{1}{2}},$$

$$z^A = -i \left(\frac{-q_0 q_A^2}{f(q_1, \ldots, q_n)}\right)^{\frac{1}{2}}$$ finite on horizon for $B_0 \neq 0$ (non-ext solns).

We have set $a_0 = \omega = B_0$ in above to get regular solns (finite $s$)
We have set $\beta = 0$ s.t. asymptotic region at $\tau = 0$. Then scale $\tau$ s.t. $\alpha = 1$.
$\exp(b_0)$ fixed to be fn of fluxes in order to satisfy EoMs.

**Leaves a family of solns parameterised by $B_0$ and $h_0$.**

# Change of Coordinates

It's convenient to change to the radial coordinate $\rho$ given by

$$e^{-2B_0\tau} = 1 - \frac{2B_0}{\rho} = W(\rho)$$

The scalars become

$$q_0 = \pm\frac{\mathcal{H}_0}{W^{\frac{1}{2}}}, \qquad q_A = \pm\frac{3}{8ng_A}(\rho W)^{-\frac{1}{2}} \quad \text{with } \mathcal{H}_0(\rho) \text{ a harmonic fn.}$$

The general expression for the 4d line element is

$$ds_4^2 = -\mathcal{H}^{-\frac{1}{2}}\rho^{\frac{3}{4}}dt^2 + \mathcal{H}^{\frac{1}{2}}\rho^{-\frac{7}{4}}\frac{d\rho^2}{W} + \mathcal{H}^{\frac{1}{2}}\rho^{\frac{3}{4}}\left(dx^2 + dy^2\right)$$

where $\mathcal{H}$ is a fn of $\mathcal{H}_0, g_A$.

- This change of coordinates makes taking limits more transparent. In particular, horizon is now at $\rho = 2B_0$ and asymptotic region at $\rho \to \infty$.
- $B_0 \to 0$ reproduces extremal soln in literature ($\therefore$ $B_0$ is non-ext parameter).

# Thermodynamics

- Zooming in on near-horizon geometry, one can compute the horizon temperature and entropy density of the black brane. These are related by

$$B_0 = 2\pi s T_H$$

- We can also look at the asymptotic values of the 4d gauge fields to find the chemical potential

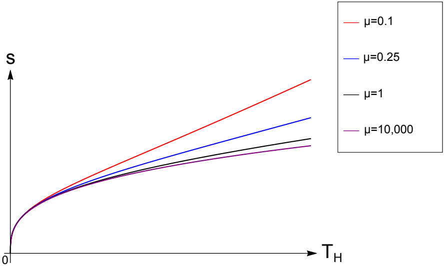
$$\mu = \frac{1}{2}\left(\frac{B_0}{Q_0}\right)\left[\coth\left(\frac{B_0 h_0}{Q_0}\right) - 1\right]$$

- Have a 2 parameter family with $(B_0, h_0)$ controlling:
  - brane geometry on gravity side
  - thermodynamic quantities $s$, $T_H$ and $\mu$ on CMT side

# Equation of State

- Can combine above expressions to find the equation of state

$$s^3 = 4\pi Z^2 T_H \left(1 + \frac{2\pi s T_H}{Q_0 \mu}\right) \qquad Z \text{ is fn of charges and fluxes.}$$




We see that $s \to 0$ as we send $T_H \to 0$ so we are justified in calling our solutions **Nernst branes**.

- Smooth crossover in behaviour from:
  - $s \sim T_H^{\frac{1}{3}}$ regime when $T_H/\mu \ll 1$
  - $s \sim T_H$ regime when $T_H/\mu \gg 1$.

# Family of Black Branes

- We now have a 2 parameter family of solutions.
- Changing $B_0, h_0$ (resp. $T_H, \mu$) changes the scalar fields and thus the 4d metric.
- We therefore need to consider 4 cases depending on whether these parameters are zero or not.
- We shall consider the near-horizon and asymptotic geometries (IR and UV of field theory) of all 4 cases.
- In each case, the geometry will turn out to belong to the so-called hyperscaling-violating Lifshitz geometries.

# hvLif Spacetimes

$$ds^2_{d+2} = r^{-\frac{2(d-\theta)}{d}}\left(-r^{-2(z-1)}dt^2 + dr^2 + dx_i^2\right)$$

$i = 1, \ldots, d$ labels spatial directions on boundary. For us, $d = 2$.
$z$ is Lifshitz exponent
$\theta$ is hyperscaling violating exponent
$(z, \theta) = (1, 0)$ returns $\text{AdS}_{d+2}$

Don't worry about not having asymptotically AdS solns.
Recently there has been much work on hvLif holography:

Under $t \to \lambda^z t, r \to \lambda r, x_i \to \lambda x_i$ we find $ds \to \lambda^{\frac{\theta}{d}} ds$.

Many CMTs have such anistropic scaling ($z \neq 1$) because they're nonrelativistic and break Lorentz symmetry.

$\theta \neq 0$ implies scale invariance of $T_{\mu\nu}$ for dual CMT is broken.

# $B_0 = 0$, $h_0 = 0$: Ground State

Set $B_0 = 0$ (extremal) and $h_0 = 0$. This means:

- $T_H = 0$
- $\mu_{\text{ext}} = \frac{1}{2h_0} \xrightarrow{h_0 \to 0} \infty$

Furthermore,

- metric becomes globally hvLif with $(z, \theta) = (3, 1)$.
- not geodesically complete
- $z^A \sim \rho^{-1/4}$ run to zero or infinity in asymptotic regions
- similar to some domain wall solns in gSUGRA which, as most symmetric solns, are interpreted as ground states.

Therefore, we interpret this solution as gravitational **ground state** of given charge sector ($Q_0 \neq 0$).
Expect to be dual to $(2+1)$-d QFT with $\theta = 1$ (hidden Fermi surfaces).

# $B_0 = 0, h_0 \neq 0$

Set $B_0 = 0$ (still extremal) but now with $h_0 \neq 0$. This means:

- $T_H = 0$
- $\mu_{\text{ext}} = \frac{1}{2h_0}$ finite

The solution interpolates between:

- hvLif with $(z, \theta) = (3, 1)$ near horizon.
- hvLif with $(z, \theta) = (1, -1)$ at infinity (conformal AdS).

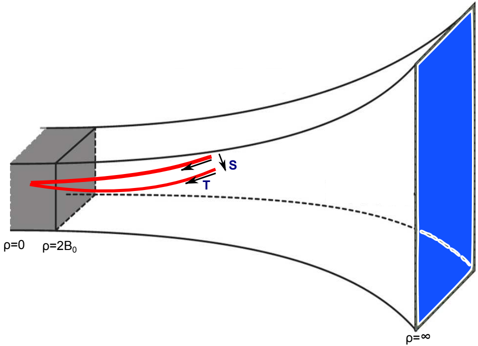
This is the extremal Nernst brane solution of Cardoso et al.

Note the effect of taking the limit $h_0 \to 0$ is to change asymptotic geometry from $(1, -1) \mapsto (3, 1)$.

We will see shortly that the $B_0 \to 0$ limit controls a change in near horizon geometry.

# Infinite Tidal Forces

The extremal ($B_0 = 0$) solns exhibit mild singular behaviour.
All curvature invariants finite as $\rho \to 0 \Rightarrow$ no curvature singularity.
The singular behaviour in question is less severe:




In hvLif spacetime, geodesic acceleration is

$$\nabla_T \nabla_T S = R(S, T)T$$

with

$$R(S, T) \sim \frac{z-1}{\rho^{2z}}$$

For $z = 1$ (AdS), $R(S, T) = 0$ so geodesics remain parallel.

Our extremal solns have $z = 3$ near horizon so $R(S, T) \to \infty$ as $\rho \to 0$.

# Spaghettification

Such infinite tidal forces result in so-called **"spaghettification"** of infalling observers due to horizontal compression and vertical elongation.

- Infinite tidal forces occur for $z \neq 1$ as $\rho \to 0$ (ext. horizon)
- Furthermore, can easily show physical 4d scalars, $z^A \sim \rho^{-1/4}$, blow up on horizon in extremal ($B_0 = 0$) case.
- Field theory can't be trusted $\Rightarrow$ study non-ext solns near horizon.
- For non-extremal solns, horizon is located at $\rho = 2B_0$. This protects them from singular behaviour which occurs behind the horizon.
- Tidal forces could still be large in low temp non-ext case but should be able to identify trustworthy range in parameter space where mapping to CMT might be possible.

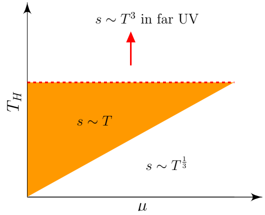
# $B_0 \neq 0$: Non-extremal Black Branes

$B_0 \neq 0$ means $T_H \neq 0$ and $\mu = \frac{1}{2}\left(\frac{B_0}{Q_0}\right)\left[\coth\left(\frac{B_0 h_0}{Q_0}\right) - 1\right]$ as before.
The 2 cases to consider are:

- $h_0 = 0$: Finite temperature and infinite chemical potential. Near horizon Rindler geometry with $(z, \theta) = (0, 2)$. hvLif with $(z, \theta) = (3, 1)$ at infinity.
- $h_0 \neq 0$: Finite temperature and finite chemical potential. Near horizon Rindler geometry again. hvLif with $(z, \theta) = (1, -1)$ at infinity.
- $B_0 \to 0$ limit changes near horizon geometry from $(0, 2) \mapsto (3, 1)$.

All values of $d, z, \theta$ from both extremal and non-extremal cases are compatible with Null Energy Condition giving a causal field theory **[Hoyos, Koroteev,** 1007.1428**]**.

# Phase Diagram




By analysing the equation of state for our gravity soln,

$$s^3 = 4\pi Z^2 T_H \left(1 + \frac{2\pi s T_H}{Q_0 \mu}\right),$$

we obtain the phase diagram for the field theory.

Scaling argument $\Rightarrow s \sim T^{\frac{d-\theta}{z}}$ for field theory:

- Non-ext Nernst brane with $B_0 \neq 0, h_0 = 0$ is dual to $(2+1)$-d QFT with $(z, \theta) = (3, 1)$ as scaling behaviour matches.
- $B_0 \neq 0, h_0 \neq 0$:
  - smooth crossover between two $(2+1)$-d QFTs: one with $(z, \theta) = (1, -1)$ in UV and one with $(z, \theta) = (0, 2)$ in IR.
  - UV scaling behaviours don't match. Along with $z^A \sim \rho^{1/4} \to \infty$, this suggests SUGRA incomplete in UV.
- If UV geometry correctly captures thermodynamic behaviour then scaling behaviour should be $s \sim T^3$

# Conclusion

- New technique for finding non-extremal black branes in gSUGRA using dimensional reduction and real formulation of special geometry.
- Family of non-extremal black branes whose entropy density vanishes in extremal limit. These are **Nernst branes**.
- Should be holographically useful as they're dual to field theories with finite temperature and chemical potential that satisfy 3rd Law.
- Analytically find solutions which interpolate between two hvLif geometries. Family is parametrised by $B_0$ and $h_0$, or equivalently, by temperature $T_H$ and chemical potential $\mu$ of the solution.
- $B_0 \to 0$ changes near-horizon geometry.
  $h_0 \to 0$ changes asymptotic geometry.
- So far solutions interpolating between hvLif geometries have relied on a mixture of analytical and numerical methods.

# Conclusion

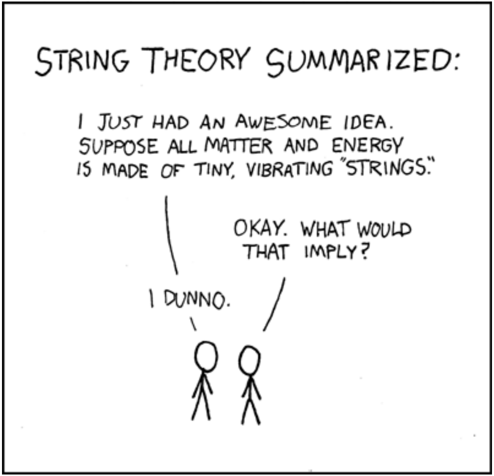
- Approached this very much from the gravity side and leave searching for concrete holographic duals to future work.
- Some of our solutions give hvLif geometries with $\theta = 1$. These lie in class of models with $\theta = d - 1$ which are thought to be dual to hidden Fermi surfaces and are some of the best studied examples in hvLif holography.
- Expect our systematic methods and analytical results that satisfy Nernst Law, can be used to make a valuable contribution to classification of solns in the rapidly increasing hvLif/CFT landscape.

# Outlook

1. 5d lift:
   - SUGRA theory not UV complete suggesting additional d.o.f. become relevant.
   - Interpret UV behaviour as decompactification limit.
   - Should embed theory into 5d gSUGRA (v. special $F(X)$).
   - Evidence suggests that dim red of theories with $AdS_d$ vacua result in hvLif$_{d-1}$ geometries
   - Expect to obtain asymptotically $AdS_5$ solns that also satisfy Nernst Law.
   - Hopefully get a clearer holographic picture using $AdS_5/CFT_4$ correspondence.
   - N.B. Asymptotic $AdS_5$ has $z = 1, \theta = 0, d = 3$ giving $s \sim T^3$ which matches proposed UV theory.
2. Dyonic charges and quantum phase transitions.

# Outlook

Xkcd says:



# Outlook

David Tong says: