"Pushing and pinching charge at strong coupling"

Talk at Univ. of Liverpool

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Based on:
1311.3292, 1401.5077, 1406.4742 with J. P. Gauntlett
1406.1659 with M. Blake
1412.2003 with M. Blake and D. Tong

Outline

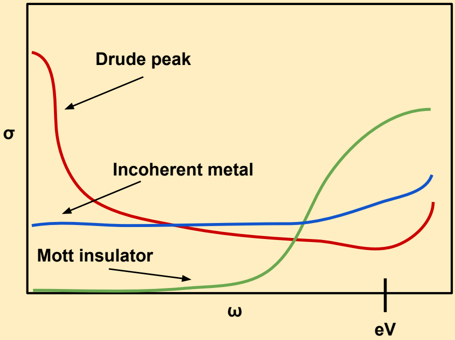
Outline

Introduction/Motivation

Some fun homework for the holographista:

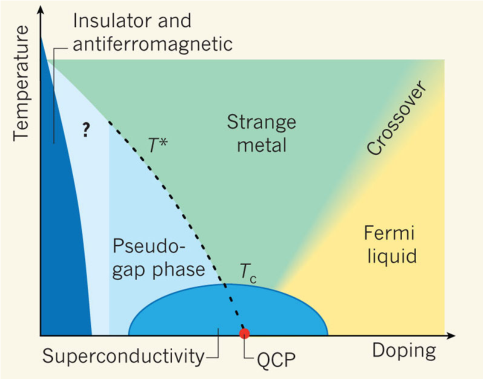
- Part I
  - Incoherent transport
  - Anomalous scaling of Hall angle
- Part II
  - Charge screening in holographic theories

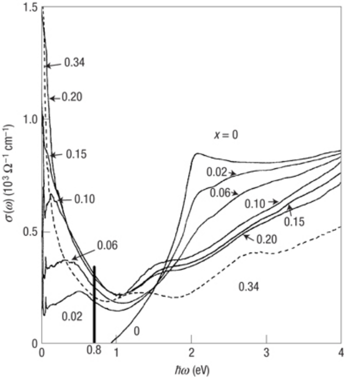
Charge transport in real materials



- Materials with charged d.o.f. can be
  - Coherent metals with a well defined Drude peak
  - Insulators
  - Incoherent conductors of electricity
- Interactions expected to become important in the incoherent phase → Possible description in AdS/CFT?

The Cuprates





The Cuprates are real life example of :

- Incoherent transport
- Anomalous scaling of conductivity and Hall angle with T

$$\rho_{DC} \propto T, \quad \theta_H \propto T^{-2}$$

Anomalous Hall angle scaling

- Introducing a magnetic field B results in currents in two directions J_x and J_y.
- There is σ_{xx} and σ_{yx}
- Hall angle is $\theta_H = \sigma_{xy}/\sigma_{xx}$
- Fermi liquids + lattice Umklapp scattering lead to

$$\sigma_{DC}^{B=0} \sim T^{-2}, \quad \theta_H \sim T^{-2}$$

- More generally, slow momentum relaxation predicts $\sigma_{DC}^{B=0}$ and θ_H scale the same way with temperature
  [Hartnoll, Kovtun, Muller, Sachdev]
- Strange metals surprisingly have

$$\sigma_{DC}^{B=0} \sim T^{-1}, \quad \theta_H \sim T^{-2}$$

- Holography evades that? Yes!

Outline

AdS/CMT

The recipe says:

Field Theory	**Bulk**
Start with CFT_d	AdS_{d+1} Asymptotics
Chemical potential μ	$U(1)$ electric charge
Finite T	Killing horizon
2-point function $G_{JJ}(\omega)$	Bulk perturbation $\delta A_x, \ldots$

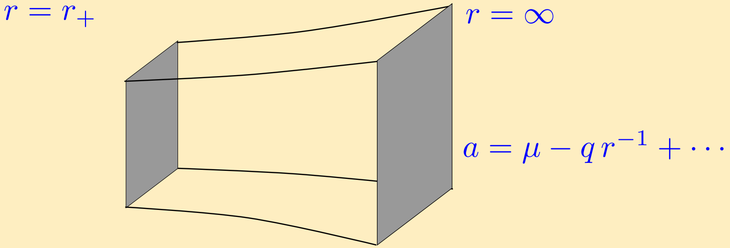
Use Kubo's formula

$$\sigma(\omega) = \frac{G_{JJ}(\omega)}{i\omega}$$

Perfect Holographic Conductor

Do it in $D = 4$ Einstein-Maxwell with AdS asymptotics:

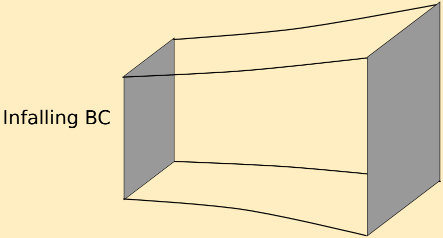
$$\mathcal{L}_{EM} = R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + 12$$
$$ds_4^2 = -U(r)\, dt^2 + U(r)^{-1}\, dr^2 + r^2 \left(dx_1^2 + dx_2^2\right)$$
$$A = a(r)\, dt$$



Background black hole has temperature T , energy E, pressure P, entropy s and charge q.

Perfect Holographic Conductor

- To calculate conductivity need to source $\delta A_x = -e^{-\imath\omega t}\frac{E_x}{\imath\omega}$ on the boundary
- Momentum (δg_{tx}) couples because of background charge



$$\delta A_x = e^{-\imath\omega t}\left(\frac{E_x}{\imath\omega} - j\,r^{-1} + \cdots\right)$$

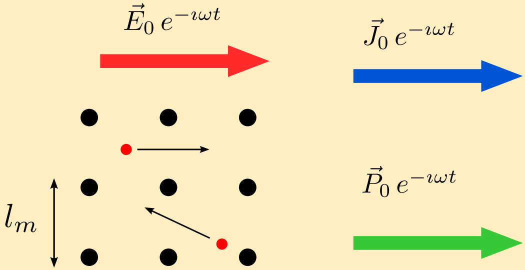
$$\omega << \mu \Rightarrow \sigma = j/E_x = \frac{\imath}{\omega}\frac{q^2}{E+P} + \frac{(T\,s)^2}{(E+P)^2}$$

[Hartnoll, Herzog]

Conserved momentum $\rightarrow$ Infinite DC conductivity $\rightarrow$ Explicitly break translations on the boundary theory

Classical Drude model

(Missing) Physics at $\omega << \mu$



- Average momentum obeys

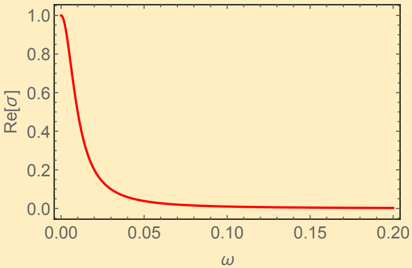
$$\langle \dot{p} \rangle = qE - \frac{1}{\tau} \langle p \rangle \Rightarrow \qquad \langle p \rangle_0 = qE \frac{\tau}{1 - \imath\omega\tau}$$

$$J = nq\frac{\langle p \rangle}{m} \Rightarrow J = \sigma\, E \Rightarrow \qquad \sigma = \frac{nq^2}{m} \frac{\tau}{1 - \imath\omega\tau}$$

- Without collisions $\tau \to \infty \; \Rightarrow \sigma = \frac{nq^2}{m} \left(\delta(\omega) + \frac{\imath}{\omega} \right)$

Classical Drude model

This is how it looks like



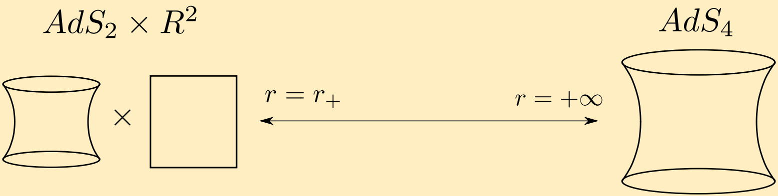
Fourier/Ohm law

- Apart from electric currents one also has a thermal current Q
- More generally, transport coefficients are packaged in a matrix

$$\left(\begin{array}{c} J \\ Q \end{array} \right) = \left(\begin{array}{cc} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{array} \right) \left(\begin{array}{c} E \\ -(\nabla T)/T \end{array} \right)$$

- With ∇T a temperature gradient

Holographic Lattice



To add momentum dissipation introduce a UV - IR benign lattice:

- Keep UV fixed point $\Rightarrow$ relevant deformation $\mathcal{O}(x)$
- Drude physics $\Rightarrow$ $T = 0$ horizon restores translations
- Charge density is a universal relevant operator $\Rightarrow$ Impose $A_t = \mu(x) - J^t(x)\, r^{-1} + \cdots$
  [Hartnoll, Hofman][Horowitz, Santos, Tong]
  $$\mu(x) = \mu_0 + A(x), \quad \langle A \rangle_L = 0$$
- $\mu_0 \Rightarrow$ chemical potential, $A'(x) \Rightarrow$ periodic electric field

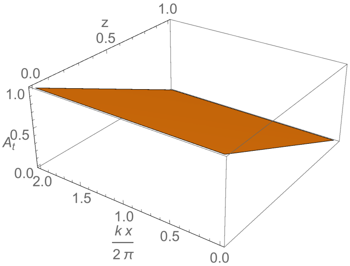
Inhomogeneous Lattices

The task is:

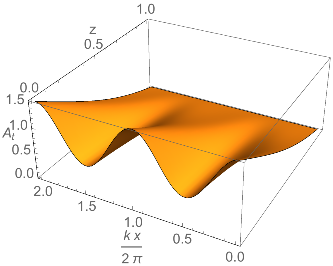
1) Solve elliptic non-linear PDEs to find background rippled black holes

2) Solve non-elliptic linear PDEs to find perturbations around numerical background to extract conductivity

[Horowitz, Santos, Tong] [D&G]



$\Rightarrow$



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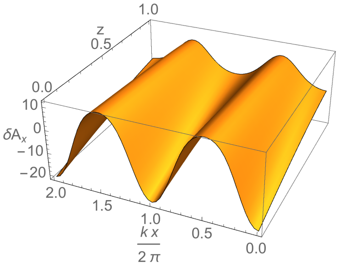
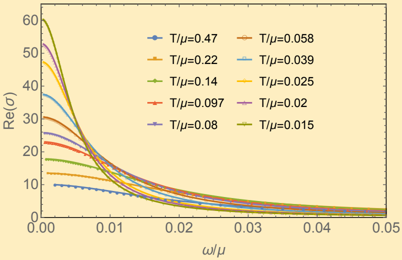
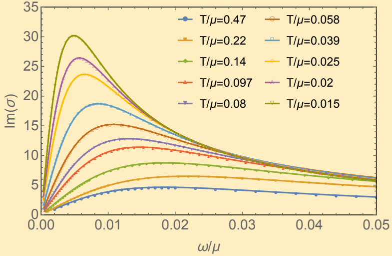


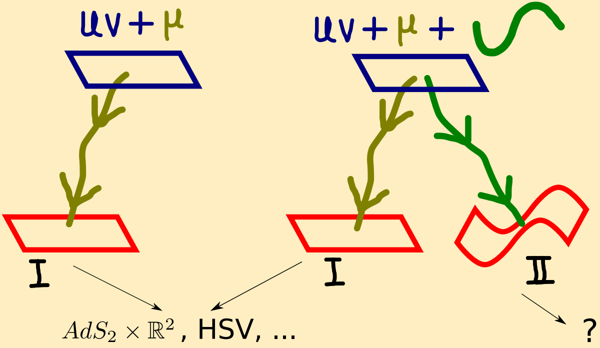
Inhomogeneous Lattices





- Our Drude peaks are there
- Nice, now get rid of them!

RG/Holographic picture



- I Charge dominated RG flows, translations restored in IR $\rightarrow$ Coherent transport
- II Lattice (+charge) dominated RG flows, translations broken in IR $\rightarrow$ incoherent transport

  [AD, Hartnoll] [AD, Gauntlett]

Q-lattices

Consider a simple model with a global $U(1)$ in addition to the gauged one [AD, Gauntlett]

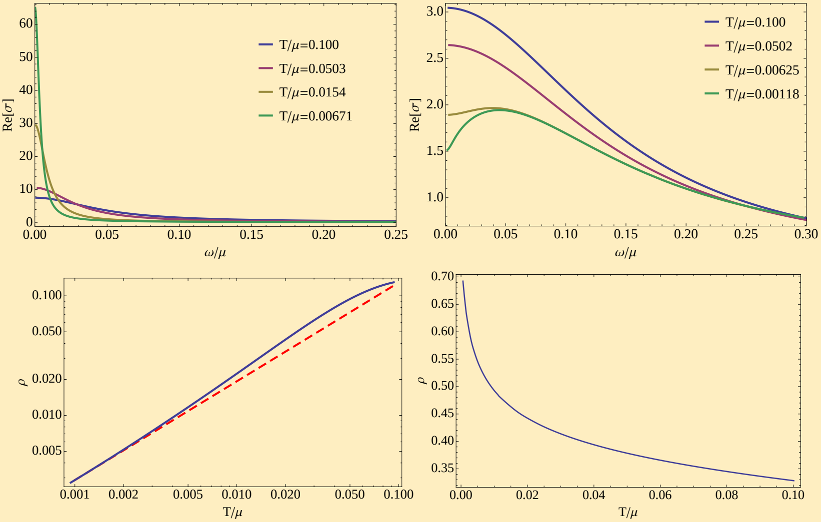
$$S = \int d^4x \sqrt{-g} \left[R + 6 - \frac{1}{4} F^2 - |\partial\phi|^2 - m^2 \, |\phi|^2 \right]$$

along with the ansatz

$$ds^2 = -U(r)\, dt^2 + U(r)^{-1}\, dr^2 + e^{2V_1(r)}\, dx_1^2 + e^{2V_2(r)}\, dx_2^2$$
$$A = a(r)\, dt, \qquad \phi = e^{\imath k x_1}\, \varphi(r)$$

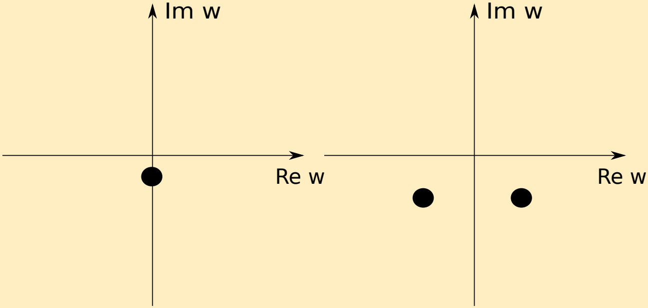
- x_1 dependence drops out due to global $U(1)$
- Leads to ODEs both for background and perturbation
- Significant simplification
- Two real scalars with $\mathcal{O}_1 \sim \cos(kx)$, $\mathcal{O}_2 \sim \sin(kx)$

Conductivity from Q-lattices



- Can model Metal - Insulator transitions
- Holography can describe incoherent transport!

RG/Holographic Picture



- QNM on the axis → coherent transport
- QNM off the axis → incoherent transport
- Recent check and also transition T [Davison, Gouteraux]

More general Q-lattices

Consider a more general situation where the instead of $\mathbb{C}^2$

$$S = \int d^4x\sqrt{-g}\left[R - V(|z|) - \frac{1}{4}Z(|z|)F^2 - G(|z|)|\partial z|^2\right]$$

- Slightly more general setup
- Imagine some complex target instead of $\mathbb{C}$
  $G(|z|)|dz|^2 \sim d\phi^2 + f(\phi)\, d\chi^2$
- The phase is compact and translations are broken by setting
  $\chi = k\, x_1$
- Closely related models with axions
  [Andrade, Withers]

More general Q-lattices

A polar decomposition yields

$$\begin{aligned}\mathcal{L} =& R - \frac{1}{2}\left[(\partial\varphi)^2 + \Phi_1(\varphi)\,(\partial\chi_1)^2 + \Phi_2(\varphi)\,(\partial\chi_2)^2\right] \\ &+ V(\varphi) - \frac{Z(\varphi)}{4}\,F^2\end{aligned}$$

The background ansatz in this notation just reduces to

$$\begin{aligned}ds_4^2 &= -U(r)\,dt^2 + U(r)^{-1}\,dr^2 + e^{2\,V_1(r)}\,dx_1^2 + e^{2\,V_2(r)}\,dx_2^2 \\ A &= a(r)\,dt, \quad \phi = \phi(r) \\ \chi_1 &= k_1\,x_1, \quad \chi_2 = k_2\,x_2\end{aligned}$$

Metallic - Insulating ground states

Imagining situations where

$$\Phi_i(\varphi) \sim e^{\delta_i \varphi}, \quad Z \sim e^{\gamma\varphi}, \quad V \sim e^{\alpha\varphi}$$

[AD, Gauntlett][Gouteraux]

$$ds^2 = -r^u\,dt^2 + r^{-u}\,dr^2 + r^{v_1}\,dx_1^2 + r^{v_2}\,dx_2^2$$
$$\phi = -\kappa\,\ln r, \quad A = r^a\,dt, \quad \chi_1 = k_1\,x_1, \quad \chi_2 = k_2\,x_2$$

- Exponents u, v_1, ... fixed by Lagrangian parameters δ_i, γ, α.
- Use perturbative argument to find small T behaviour on the horizon e.g.

$$\phi_{r=r_+} \sim -\kappa'\,\ln T, \quad s \sim T^\lambda$$

- So what?

Ohm/Fourier Law

More generally, combine E with thermal gradient ∇T to describe thermoelectric effect

$$\begin{pmatrix} J \\ Q \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} E \\ -(\nabla T)/T \end{pmatrix}$$

- Analytic argument to express DC transport coefficients in terms of bh horizon data

$$\sigma_{DC} = \left[\frac{Z(\phi) s}{4\pi e^{2V_1}} + \frac{4\pi q^2}{{k_1}^2 \Phi_1(\phi) s} \right]_{r=r_+} = \sigma_{ccs} + \sigma_{dis}$$

$$\bar{\kappa}_{DC} = \left[\frac{4\pi s T}{{k_1}^2 \Phi_1(\phi)} \right]_{r=r_+}, \quad \alpha_{DC} = \bar{\alpha}_{DC} = \left[\frac{4\pi q}{{k_1}^2 \Phi_1(\phi)} \right]_{r=r_+}$$

- Also possible for inhomogeneous lattices [AD, Gauntlett]

New insight from Holography

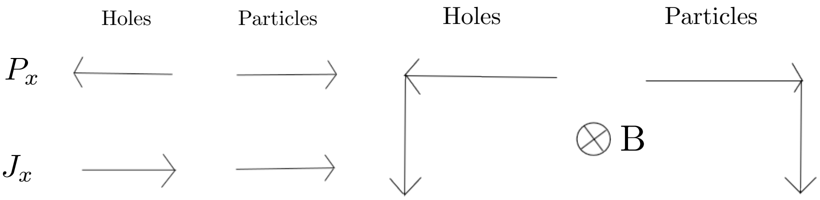
Two terms of σ_{DC} come from different physics!

- Fix a combination of E and ∇T such that we have no heat current
- In this situation we still have finite electric current

$$\sigma_{Q=0} = \sigma - \frac{\alpha\bar{\alpha}T}{\bar{\kappa}} \Rightarrow$$
$$\sigma_{Q=0} = \sigma_{ccs} = \left[\frac{Z(\phi)s}{4\pi e^{2V_1}}\right]_{r=r_+}$$

- Has to come from evolution of neutral pairs
- This contribution is exponentially suppressed in DC transport for Fermi liquids!
- Low T behaviour of σ_{DC} can be determined by either σ_{ccs} or σ_{dis}

Hall angle [AD, Blake]



Weak coupling fantasy!

- Particles and holes deflected in the same direction
- Opposite charge $\Rightarrow$ Don't contribute to J_y
- Expect dissipative component of the current to dominate Hall angle

Hall angle [AD, Blake]

Same model, same ansatz with $B \neq 0$ this time lead to

$$\sigma_{xx} = \frac{e^{2V} k^2 \Phi (B^2 Z^2 + q^2 + Z e^{2V} k^2 \Phi)}{(B^2 Z + e^{2V} k^2 \Phi)^2 + B^2 q^2}\Bigg|_{r_+}$$

$$\sigma_{xy} = \frac{Bq (B^2 Z^2 + q^2 + 2 Z e^{2V} k^2 \Phi))}{(B^2 Z + e^{2V} k^2 \Phi)^2 + B^2 q^2}\Bigg|_{r_+}$$

A bit of an ugly mess but...

$$\theta_H = \frac{Bq}{e^{2V} k^2 \Phi}\left[\frac{B^2 Z^2 + q^2 + 2 Z e^{2V} k^2 \Phi}{B^2 Z^2 + q^2 + Z e^{2V} k^2 \Phi}\right]\Bigg|_{r_+}$$

$$= \frac{Bq}{s\, k^2 \Phi}\, \mathcal{W} = \frac{q^2}{k^2\, s\, \Phi}\, \frac{B}{q}\, \mathcal{W}$$

- Notice $1 < \mathcal{W} < 2$

Hall angle [AD, Blake]

For $B^{1/2} << T << \mu$

$$\theta_H \propto \frac{B}{q}\,\sigma_{dis}^{B=0}$$
$$\sigma_{DC}^{B=0} = \sigma_{ccs}^{B=0} + \sigma_{dis}^{B=0}$$

- The Hall angle scaling with T can be independent how $\sigma_{DC}^{B=0}$ scales if $\sigma_{ccs}^{B=0}$ dominates
- Can't have this with weakly coupled Fermions in DC conductivity. Particle-hole creation is gapped at low energies.

Outline

Charge Screening

- How does a point-like object affect a uniform charge distribution?
- Particle statistics/Interactions leave imprint on response
  - Debye - Hückel model
  - Lindhard theory
- Holography?
  Similar sort of questions in [Horowitz, Iqbal, Santos, Way]

Debye - Hückel model

- Write equation for electric potential ϕ with sources
- Assume local thermodynamic equilibrium $\rightarrow$ Boltzmann statistics for ρ
- Good approximation at high temperatures T

$$\nabla^2\phi = -\left(Q\,\delta^3(r) - q\,\rho_0 + q\,\rho(r)\right)$$

$$\rho(r) = \rho_0\,e^{-q\phi(r)/k_BT} \approx \rho_0\,\left(1 - q\phi(r)/k_BT\right)$$

$$\left(\nabla^2 - \lambda_D^{-2}\right)\phi = -Q\,\delta(r), \quad \lambda_D^2 = k_BT/q^2 n_0$$

$$\Rightarrow \phi = \frac{Q}{4\pi r}\,e^{-r/\lambda_D}$$

Lindhard Theory

- Perturb Hamiltonian by $\Delta H = q\,\phi(r)$
- States smoothly deformed $|k\rangle \to |\psi(k)\rangle$
- Statistics captured by Fermi - Dirac distribution f

$$\rho^{\mathrm{ind}}(r) = q\,g_s \int \frac{d^3k}{(2\pi)^3}\, f(k)\, \left[|\langle r|\psi(k)\rangle|^2 - |\langle r|k\rangle|^2 \right]$$

- Relevant quantity to extract is the charge susceptibility

$$\chi_Q(k) = \frac{\rho^{\mathrm{ind}}(k)}{k^2\phi(k)}$$

Linhard theory - Friedel Oscillations

- Discontinuity of f at $k = k_F$ smooths out to a $\log$ in χ_Q

$$\chi_Q(k) = \frac{k_{TF}^2}{k^2} F(k/2\,k_F)$$
$$F(x) = \frac{1}{2} + \frac{1-x^2}{4x}\,\log\left|\frac{x+1}{x-1}\right|$$

- Translating to real space Friedel oscillations for long distances

$$\rho^{\text{ind}} \propto r^{-3}\,\cos(2k_F r)$$

Charge Screening in Holography

- Consider a charged black hole with AdS asymptotics

$$A \approx \mu_{(0)}\, dt + \rho_{(0)}\, z\, dt + \cdots$$

- We want to introduce a (local) perturbation on the boundary

$$\mu \to \mu_{(0)} + \delta\mu(\vec{x})$$

- And read off the induced charge $\delta\rho(\vec{x})$

$\Rightarrow$ Static modes of longitudinal sector in Einstein-Maxwell

$\Rightarrow$ Work in momentum space $\delta\rho(k) \propto \chi(k)\, \delta\mu(k)$

Charge Screening in Holography

Cases to consider:

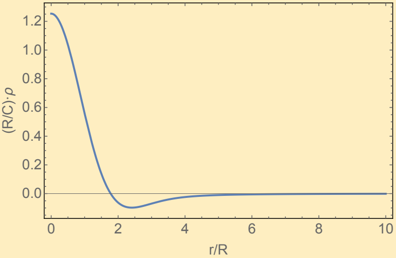
- $\mu = 0$, $T = 0$, i.e. AdS_4
- $\mu = 0$, $T > 0$, i.e. Sch. black brane
- $\mu \neq 0$, $T > 0$, i.e. RN black brane
- $\mu \neq 0$, $T = 0$, i.e. extremal RN

Charge Screening in Holography

- For $\mu = 0$, $T = 0$ scale invariance implies $\chi(k) = k$
- Can also see from exact bulk perturbation
  [Chesler, Lucas, Sachdev]

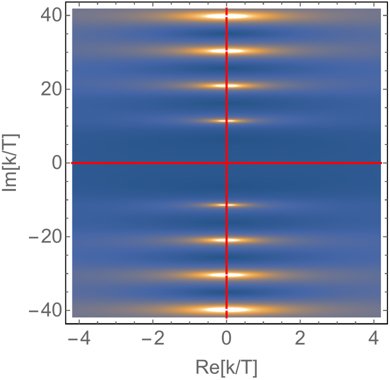
$$ds_4^2 = z^{-2}\left(-dt^2 + dx_1^2 + dx_2^2 + dz^2\right)$$
$$\delta A_t = e^{\imath\vec{k}\cdot\vec{x}} e^{-|\vec{k}|\,z} \to e^{\imath\vec{k}\cdot x}\left(1 - |\vec{k}|\,z + \cdots\right)$$

- For a Gaussian source $\delta\mu(r) = C\,e^{-r^2/2R^2}$ this gives



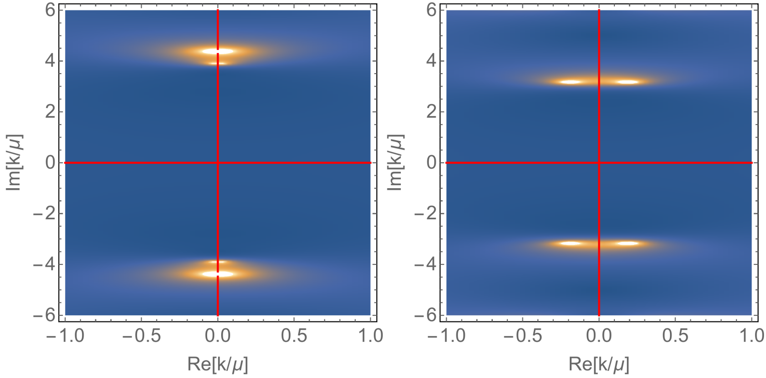
Charge screening in holography

- For $\mu = 0$, $T > 0$ need to do numerics
- Long range behaviour captured by analytic structure of $\chi(k)$ in complex $k-$plane
- Poles on Im axis give exponential fall off $\delta\rho \approx r^{-1/2} e^{-\mathrm{Im}k\, r}$



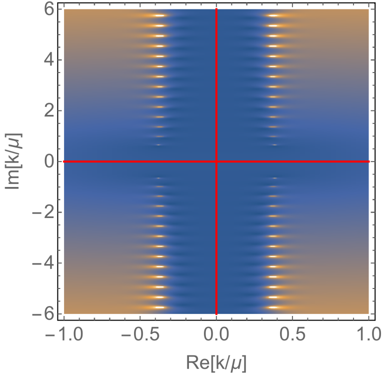
Charge screening in holography

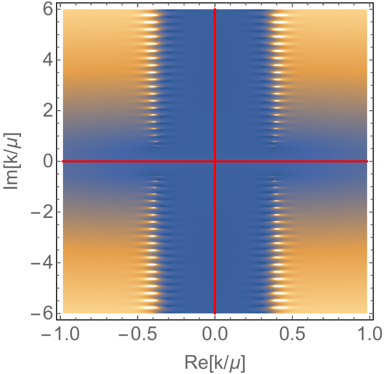
- For $\mu \neq 0$, $T >> \mu$ looks similar to $\mu = 0$ case
- There is a $T_c \approx .33\mu$ where two poles acquire non-zero real parts!
- This is when charge oscillations happen $\delta\rho \propto e^{-\lambda_1 r} \cos(\lambda_2 r)/\sqrt{r}$



Charge screening in holography

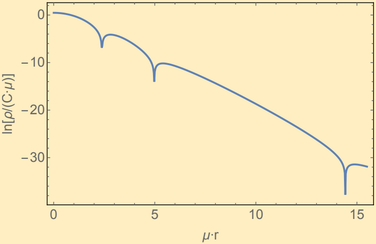
- At $T << \mu$ more poles coalesce to branch cuts
- They end at $k_*/\mu_0 = \pm 2^{-3/2} \pm \imath/2$
- Charge oscillations remain exponentially damped!

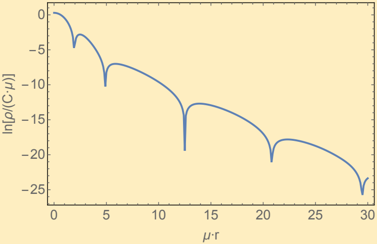




Charge screening in holography

Charge oscillations in coordinate space for Gaussian source at $T_c > T > 0$ and $T = 0$





Distance between nodes matches with $\frac{\pi}{\mathrm{Re}k_*}$ of leading pole

- Charge oscillations present in holography but not quite Friedel. Result of strong coupling or large N?
- Reasonable to expect that from field theory? e.g. $\mathcal{N} = 4$

Outline

Summary / Outlook

- Holography is a good tool to study transport in strongly coupled systems
- No assumption of quasiparticles
- Offers new insight for real world problems
- Understand better the physics of the new ground states
- Friedel oscillations with exponential damping at strong strong coupling (or large N?)
- DC transport from bh horizons: General statement?