PROCEEDINGS OF SCIENCE

Phenomenology of static-light mesons from unquenched lattice QCD calculations.

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We present important results for the static-light meson from unquenched lattice QCD. The unquenched gauge configurations were generated using the non-perturbatively improved clover action. At the fixed lattice spacing of 0.1 fm the lightest sea quark mass used is a third of the strange quark mass. A comparison is made between heavy-light chiral perturbation theory and the f_B^{static} decay constant. The mass of the bottom quark is also reported: $\overline{m_b}(\overline{m_b}) = 4.25(2)(11)$ GeV, where the first error is statistical and the last error is the systematic uncertainty.

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1. INTRODUCTION

There are a number of important issues in heavy quark phenomenology that can be addressed using the static quark formulation for the heavy quark. The mass of the bottom quark is a fundamental parameter of the standard model. As noted by Gimenez et al [1], the error on the mass of the bottom quark due to the use of static quarks for the b quark is only of order 30 MeV [1].

The heavy flavour experiments such as BaBar, BELLE, CDF, and D0 are extensively testing the CKM matrix formalism. There are a number of non-perturbative QCD matrix elements that must also be computed to test the CKM formalism. In particular, the ratio of the decay constants of the B_s to B mesons ($\frac{f_{B_s}}{f_B}$) is a crucial QCD quantity for the unitarity checks of the CKM matrix.

2. COMPUTATIONAL DETAILS

The basis of our calculation is unquenched gauge configurations generated with the non-perturbatively improved clover action and the Wilson gauge action. The lattice parameters are: volume 16^3 32, β = 5.2, and the clover coefficient was the non-perturbative value of 2.0171. The full details of the action and results on the hadron spectroscopy have been published [2].

UKQCD has already published [3] an extensive analysis of the spectrum of static-light mesons.

The f_B decay constant is extracted from the amplitudes in the two point correlator.

$$C(t) = \sum_x \langle 0 \mid A_4(x,t) \Phi_B^\dagger(x,0) \mid 0 \rangle \tag{2.1}$$

$$\to Z_L Z_{\Phi_B} \exp(-a\mathcal{E}t) \tag{2.2}$$

where Φ_B is the interpolating operator for static-light mesons. We used all-to-all propagators and fuzzed sources to get accurate correlators [3]. We fit a 3 exponential model to a 5 by 5 smearing matrix.

The f_B decay constant is defined by the matrix element below:

$$\langle 0 \mid A_\mu \mid B(p) \rangle = i p_\mu f_B \tag{2.3}$$

The axial current is improved using the ALPHA formulation [4], including the correction term introduced by Morningstar and Shigemitsu [5]. The connection between the decay constant and the amplitude is.

$$f_B^{static} = Z_L \sqrt{\frac{2}{M_B}} Z_A^{static} \tag{2.4}$$

where Z_A^{static} is the renormalisation factor. Here we explore the sea quark dependence of Z_L. We have checked that the variation of Z_A^{static} with sea quark mass is small.

3. THE MASS OF THE BOTTOM QUARK

Full details of our calculation of the bottom quark mass have been reported in [6].

The parameters $\kappa_{sea} = 0.1355$ $\kappa_{val} = 0.1350$ were used for the central value. The quantity $\mathcal{E}$, from the lattice calculation, contains an unphysical $\frac{1}{a}$ divergence (δm) that must be subtracted off to obtain the physical binding energy (Λ_{static}).

$$\Lambda_{static} = \mathcal{E} - \delta m \tag{3.1}$$

The pole quark mass is determined from

$$m_b^{pole} = M_{B_s} - \Lambda_{static} \tag{3.2}$$

The physical value [7] of the meson mass M_{B_s} (5.369 GeV) is used.

In the static theory δm has been calculated to two loops by Martinelli and Sachrajda [8]. The pole mass is converted to $\overline{MS}$ using continuum perturbation theory [9].

$$m_b^{\overline{MS}}(\mu) = Z_{pm}(\mu) m_b^{pole} + O(1/m_b) \tag{3.3}$$

The lattice matching is only done to $O(\alpha^2)$, hence we convert the pole mass to $\overline{MS}$ at the same order, using a consistent coupling, so the differences in the series are physical. This avoids problems with renormalons.

Our final result for $\overline{m_b}(\overline{m_b})$ (in GeV) is

$$4.25 \pm 0.02 \pm 0.03 \pm 0.03 \pm 0.08 \pm 0.06 \tag{3.4}$$

where the errors are (from left to right): statistical, perturbative, neglect of $1/m_b$ terms, ambiguities in the choice of lattice spacing, and error in the choice of the mass of the strange quark.

Our result is consistent with that from Gimenez et al. [1] $(\bar{m}_b(\bar{m}_b) = 4.26 \pm 0.09 \text{ GeV})$ from an unquenched QCD calculation.

4. CHIRAL LOGS IN THE HEAVY-LIGHT DECAY CONSTANT

The error on the ratio of the $\frac{f_{B_s}}{f_B}$ has recently been increased, because the chiral log term has not been observed in lattice data [10]. For example, the JLQCD [11] collaboration quote $f_{B_s}/f_{B_d} = 1.13(3)(^{+13}_{-\ 2})$, where the first error is statistical and the second error is from the systematic uncertainties. The dominant systematic uncertainty in JLQCD's result is from the chiral extrapolation. All lattice calculations, apart from some preliminary evidence from one [12], have only seen linear dependence of the heavy-light decay constant on the quark mass.

The one loop correction, in heavy-light chiral perturbation theory, to the static-light decay constant is [11]

$$\frac{\Phi_{f_{B_d}}}{\Phi_{f^0_{B_d}}} = 1 - \frac{3(1+3g^2)}{4} \frac{m_\pi^2}{(4\pi f)^2} \log(\frac{m_\pi^2}{\mu^2}) \tag{4.1}$$

where g is the $B^*B\pi$ coupling and $\Phi_{f_{B_d}} \equiv f_{B_d}\sqrt{M_{B_d}}$. The coupling g has recently been measured at CLEO. It has also been determined from quenched lattice QCD.

In figure 1 Z_L using κ_{sea} = 0.1350, 0.1355, and 0.1358 is plotted. At κ_{sea} = 0.1358, 138 configurations were used. The other two κ values used ensemble sizes of 60. JLQCD [11] reported that the heavy-light decay constant had only linear dependence on quark mass. The new data at κ_{sea} = 0.1358 does show some deviation from linearity. Although the figure 1 is encouraging – the deviation is not statistically significant. The curve is the expression from chiral perturbation theory (equation 5.1) drawn for illustration, rather the result of a fit.

As has been noted by many authors [13, 14] the chiral log structure of f_π and f_B are rather similar. Hence UKQCD's claim to see the effect of the chiral logs in f_π [15] suggests that there should be deviations from linear dependence of $f_{B_s}^{static}$ on the quark mass.

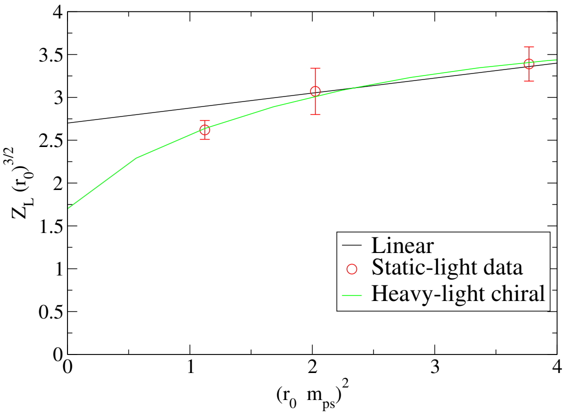

Figure 1: Static-light decay constant as a function of quark mass.

The value of the lightest pion in our calculation is roughly 420 MeV [15]. The different treatments of the heavy-light chiral perturbation theory of Sanz-Cillero et al. [16] show that a deviation of linearity in quark mass for f_B is expected at these pion masses.

In UKQCD's work on chiral logs in the pion decay constant, there was a concern about finite volume effects [15]. At $\kappa_{sea} = 0.1358$ it was argued from chiral perturbation theory that the finite volume effects were of the order of 8% in f_π. A similar order of magnitude effect was also estimated by Colangelo and Haefeli [17]. The volume of the lightest data set is $(1.5 \text{ fm})^3$. Recently Arndt and Lin [18] have studied the effect of the finite volume on the ratio of heavy light decay constants and bag parameters. For a pion mass around 400 MeV on box size of $(1.6 \text{ fm})^3$, the finite volume effect in the ratio $\frac{f_{B_s}}{f_B}$ is 0.006. This suggests that the finite size effects are small. However, the next to leading order estimate of finite size effects in f_π was significant [17]. Unfortunately, Colangelo and Haefeli [17] claim that there is not enough information to do a similar estimate for f_B.

As the aim of this work is to look for chiral logs in the f_B decay constant that are a small effect, it is important to reduce the statistical errors. We are already using all-to-all techniques and fuzzing. The number of available gauge configurations is fixed. The ALPHA collaboration [19] have developed a new variant of the static formalism that reduces the $1/a$ mass renormalisation that is thought to be the reason for the poor signal to noise ratio in static-light calculations. We are currently running with this formalism and we expect to report results in the future.

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